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Non-planar motions due to nonlinear interactions between unstable oscillatory modes in a cantilevered pipe conveying fluid

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Abstract: Dynamics of a cantilevered pipe conveying fluid has been studied for a long time. When the flow velocity exceeds a certain value, the damping ratio of a certain mode becomes negative. Long term efforts have been poured to the post-critical dynamical behavior by many researchers. As the flow velocity is increased further, another mode can also experience an oscillatory instability. In such situations, we have to consider nonlinear modal interactions to clarify the evolutions of the amplitudes of unstable modes. In this study, we consider the non-planar oscillations of a cantilevered fluid conveying pipe with an end mass. We focus on the nonlinear interactions of two unstable oscillatory modes. The amplitude equations are derived and nonlinear analyses are conducted. It is clarified that some types of non-planar motions can be produced due to nonlinear interactions of two different unstable modes. Moreover, experiments were conducted to verify the theoretical predictions. In experiments, we observed some non-planar motions that show qualitatively good agreement with the theory.

Keywords: Non-planar motion, Pipe conveying fluid, Double Hopf bifurcation

1. Introduction

High-codimensional bifurcations have attracted the interest in many researchers. Dynamics of a cantilevered pipe conveying fluid is a typical problem of dynamic instabilities in continuous systems[1]. From the linear stability analyses, in a certain parmeter regions, it is expected that double Hopf bifurcation becomes a problem. At the double Hopf bifurcating point, the linear system has two pairs of eigenvalues $\pm i\omega_m$, $\pm i\omega_n$ ($\omega_m \neq \omega_n$). In such situations, we have to consider nonlinear modal interactions to clarify the evolutions of the amplitudes of unstable modes. In the prevoius study[2], we consider the Hopf-Hopf interactions of unstable modes in a plane. In this study, we consider the non-planar oscillations of a cantilevered fluid conveying pipe with an end mass. We focus on the nonlinear interactions of two unstable oscillatory modes. The amplitude equations of non-planar mixed modal oscillations are derived from the non-selfadjoint partial differential equations and their bounday conditions. From the nonlinear analyses, it is clarified that mixed modal non-planar motions can be produced due to nonlinear interactions of two different unstable modes. Moreover, experiments were conducted to verify the theoretical predictions. In experiments, we observed some non-planar motions that show qualitatively good agreement with the theory.

2. Results and Discussion

Figure 1 shows the analytical model of non-planar self-excited pipe vibration. Pipe is hung vertically under the gravity and the lumped mass is attached at the lower free end. From the linear stability analyses, in a certain range of parameters, it is confirmed that second and third modes become simultaneously unstable. Let A_i and B_i be the complex amplitudes of the unstable two modes in X-Y plane and X-Z plane, respectively (i =2, 3). We derive evolutinal equations of copmplex amplitude equations as follows:

$$\dot{A}_2 = \left(-\omega_{2i} + \xi_{12}|A_2|^2 + \xi_{22}|A_3|^2 + \xi_{32}|B_2|^2 + \xi_{42}|B_3|^2\right)A_2 + \xi_{52}\bar{A}_2 B_2^2 + \left(\xi_{62}A_3\bar{B}_3 + \xi_{72}\bar{A}_3 B_3\right)B_2 \qquad (1)$$

where ω_{2i} is the linear damping ratio. Evolutional equations of A_3, B_2 and B_3 are writen in the similar manner. From the nonlinear analyses, it is clarified that mixed modal non-planar motions can be produced due to nonlinear interactions of two different unstable modes.

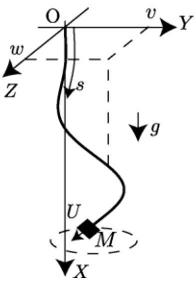

Fig. 1. Analytical model

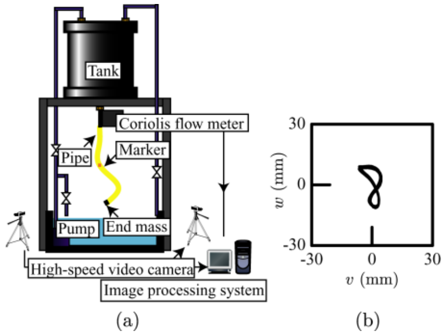

Fig. 2. (a) Experimental set-up, (b) Non-planar motion

Experiments were conducted to verify the theoretical results. Figure 2(a) shows the experimental set-up. We used the image processing system to conduct three-dimensional measurements of pipe vibrations. Figure 2(b) shows the non-planar mixed modal pipe vibrations. Frequency of v is almost twice the frequency of w. We also observed some complex non-planar motions in experiments.

3. Concluding Remarks

We consider the non-planar oscillations of a cantilevered pipe conveying fluid. In particular, we focus on the nonlinear interactions between two unstable oscillatory modes. First, we derive the complex amplitude equations. It is clarified from nonlinear analyses that mixed modal non-planar motions can be produced due to nonlinear interactions of two different unstable modes. Second, experiments were conducted to verify the theoretical results. Some non-planar motions due to two unstable modes were obserbved. Theoretical results qualitatively give a good account of the typical feqtures of non-planar mixed modal self-excited oscillations.

References

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