Nonlinear Interaction between Longitudinal and Transverse Vibrations of a Microbeam under Periodic Opto-Thermal Excitation

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Abstract: The paper investigates the nonlinear dynamics of coupled longitudinal-flexural vibrations of a microbeam clamped at both ends - the basic sensitive element of a wide class of microsensors of physical quantities - under laser opto-thermal excitation in the form of periodically generated Gaussian pulses acting on some part of the surface of the beam element. The steady-state time-harmonic temperature distribution in the volume of the beam resonator is found for a given power, duration, and periodicity of laser pulses. The modes of parametric oscillations of the microbeam are investigated under conditions of internal combinational resonance between some two flexural and lower longitudinal modes of free oscillations of the resonator.

Keywords: N/MEMS, sensor, combinational resonance, nonlinear oscillations, laser-induced vibrations

1. Introduction

The principle of laser opto-thermal action on a deformable medium is finding ever wider application in problems of non-destructive testing of equipment and structures [1,2], determination of physical and mechanical properties of materials [3,4], analysis of geometric and physical parameters of objects and structures on nano- and the microscale [5,6], in biomedicine [7], as well as in the nano and microsystems industry [8-11].

In earlier works [12-14], the dynamics and elastic stability of the N/MEMS beam element under short-term thermal effects was investigated. In those works, the stage of bending wave formation at sufficiently short times was considered. In paper [15], attention was paid to the interaction of different modes of vibration, both transverse and longitudinal, in the case of free oscillations under pulsed laser action. It was shown that the initial perturbation in the longitudinal direction can effectively excite bending vibration modes, which, on the whole, leads to a longitudinal-transverse beating regime with noticeable amplitudes. It was noted that the period and amplitude of these beats substantially depend on the frequency detuning parameter between the sum of the bending vibration frequencies and the longitudinal vibration frequency.

In this work, we study the nonlinear dynamics of coupled longitudinal-flexural vibrations of a microbeam clamped at both ends - the basic sensitive element of a wide class of microsensors of physical quantities - under laser opto-thermal excitation in the form of periodically generated Gaussian pulses acting on some part of the surface of the beam element.

2. Results and Discussion
A graphical diagram of the problem under consideration is shown in Fig. 1.

Fig. 1. Graphical scheme of the problem. \( u, \ w \) – longitudinal and transverse components of the displacements vector of the rod material point with coordinates \( x, z \) at time \( t \); \( T(x, z, t) \) – temperature deviation from the reference value; \( Q(x, z, t) \) – body heat flux simulating the thermo-optical effect of a laser on a section of a beam \( l_1 < x < l_2 \); \( k, h_1 \) – coefficients of thermal conductivity and convection with the external environment, respectively.

To describe the deformed state of the microbeam, the mechanical Bernoulli-Euler model and the model of longitudinal vibrations of the rod are used. Coupled geometrically nonlinear equations of longitudinal-flexural vibrations of a beam element under laser temperature exposure are formulated under the assumption \( u = O(w^2) \); the thermal effect of the laser on the structure is reduced to a thermal axial force \( N^T = E b \int_{-h/2}^{h/2} z T dz \) and thermal bending moment \( M^T = E b \int_{-h/2}^{h/2} z^2 T dz \) (\( E, \alpha \) – Young modulus and linear thermal expansion coefficient of the material; \( b, h \) - beam section width and height). Under the assumptions made, the dimensionless equations of motion of the beam resonator take the form

\[
\ddot{u}'' + 12\beta \dot{w}' \ddot{w}'' = 12\beta^2 \ddot{u} + \ddot{N}_x^T, \\
\dddot{w} + \ddot{w} + \dddot{N}^T \dddot{w}'' = -\dddot{M}_{xx}^T + \frac{1}{\beta} \dddot{u}' \dddot{w}'' + 6(\dddot{w}')^2 \dddot{w}'' + 12\beta \dddot{u} \dddot{w}',
\]

where \( \ddot{u}, \dddot{w} \) – dimensionless displacements normalized to the beam height \( h \); \( \beta = \frac{h}{12L} \) – dimensionless parameter, characterizing slenderness of the beam; the subscript \( x \) denotes the corresponding derivatives with respect to the longitudinal coordinate.

The steady-state time-harmonic temperature distribution in the volume of the beam resonator is found analytically for a given power, duration, and periodicity of laser pulses. Using asymptotic methods of nonlinear mechanics, the modes of parametric oscillations of a microbeam under conditions of internal combinational resonance between some two bending and lower longitudinal modes of free oscillations of the resonator are investigated.

3. Concluding Remarks

The study made it possible to show the technical feasibility of laser generation of bending vibrations of a microscale resonator using the parametric resonance mechanism and to evaluate the influence of the factor of interaction of longitudinal and bending modes of vibrations on the nonlinear dynamics of the system.

Acknowledgment: The work was supported by RFBR grant 20-01-00537.

References


