DSTA 2021 dys-ta.com
16th INTERNATIONAL CONFERENCE
Dynamical Systems – Theory and Applications
December 6-9, 2021, On-line

A 3D multiple scattering formulation to model elastic waves interacting with surface resonators

XINGBO PU*, ANTONIO PALERMO, ALESSANDRO MARZANI

Department of Civil, Chemical, Environmental and Materials Engineering, University of Bologna, 40136 Bologna, Italy
* Presenting Author: xingbo.pu2@unibo.it

Abstract: We present a closed-form formulation to model the 3D dynamic response of an elastic half-space coupled to a cluster of surface resonators when excited by a harmonic point force. The technique exploits the analytical solution of the canonical Lamb's problem in a multiple scattering methodology to formulate the wavefields generated by the resonators. For an arbitrary number of surface resonators arranged atop an elastic half-space in an arbitrary configuration, the displacement field is obtained in closed-form and validated via finite element simulations in three-dimensional contexts.

Keywords: Elastic metasurfaces, seismic metamaterials, multiple scattering, Rayleigh waves.

1. Introduction

The interaction between waves and surface resonant structures plays an important role in modelling the so-called site-city effect [1] and in designing resonant barriers for ground-borne vibration attenuation [2, 3]. Pivotal for understanding the coupled substrate-resonator response is the possibility of predicting the total wavefield given by both the propagating waves and the scattered fields generated by the resonant structures. Currently, such prediction can be achieved only in finite-size systems by exploiting computationally costly numerical techniques, like finite element (FE), since no closed-form formulations are available for this purpose.

To overcome this limitation, in this work we provide an analytical formulation to predict the wavefield of an elastic half-space coupled to a generic array of vertical surface resonators, as shown in **Fig. 1**a. The methodology exploits the classical Lamb's problem [4] to formulate the incident wavefield and its Green's functions to describe the scattered fields given by the resonators subjected to a base motion.

2. Formulation in brief

According to the multiple scattering technique, the total wavefield can be obtained as:

$$\boldsymbol{u}(\boldsymbol{r}) = \boldsymbol{u}_{inc}(\boldsymbol{r}) + \sum_{n=1}^{N} F_n\, \boldsymbol{G}(\boldsymbol{r} - \boldsymbol{R}_n) \tag{1}$$

In which $\boldsymbol{u}_{inc}(\boldsymbol{r})$ is the incident wavefield at position $\boldsymbol{r} = [x, y, z]$ generated by the harmonic point source acting at $\boldsymbol{r} = \boldsymbol{0}$, $\boldsymbol{R}_n$ denotes the resonator positions, $\boldsymbol{G}$ is the Green's function, and F_n is the normal stress provided by resonators which can be determined by boundary conditions. Once F_n is obtained, the wavefield can be calculated by substituting it into Eq. (1). The detailed procedure to calculate F_n is provided in [5].

3. Results and Discussion

We consider an elastic half-space coupled with two identical resonators. We set the material properties for the elastic half-space as: $E = 46$ MPa, $\upsilon = 0.25$, $\rho = 1800$ kg/m^3, namely Young modulus, Poisson ratio and density, respectively, and the resonators with mass and resonant frequency equal to $m_1 = m_2 = 1000$ kg and $f_{r1} = f_{r2} = 2$ Hz, respectively. The resonators are placed at $x_1 = 6$ m, $x_2 = 9$ m, $y_1 = y_2 = 0$, while a point source $Q = 1$ Pa is located at the origin of the coordinate system. **Fig. 1**b shows the variation of the vertical displacement at the resonators footprint for different frequencies of the input source. The solid and dashed lines denote the analytical results, while dots indicate the results from 3D FE simulations. An excellent agreement between the analytical and the numerical results is found. The drop in the amplitude of the vertical displacement occurs at the resonance frequency of the oscillators as a result of the interaction between the incident and scattered fields.

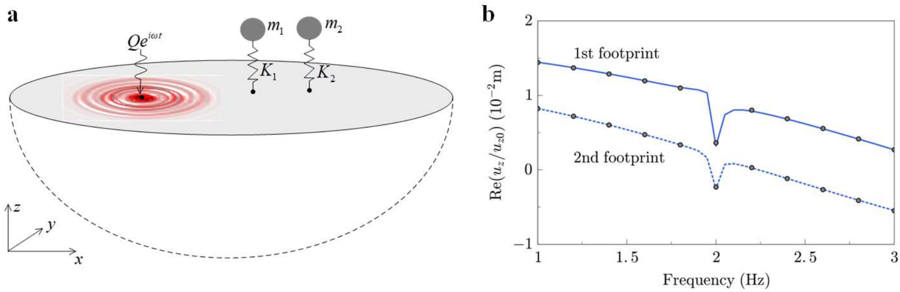

Fig. 1. (a) The schematic of the considered system. (b) The real part of the vertical displacement of resonator footprint vs. frequency. For comparison, we also provide the FE solutions denoted by dots.

4. Concluding Remarks

An analytical formulation to study the interaction of elastic waves with surface resonators is briefly discussed. The formulation makes use of the solution of the canonical Lamb's problem to calculate the incident field generated by a point source and the scattered field introduce by each resonator motion. The formulation, validated against FE simulations, can handle an arbitrary number of resonators arranged on the surface in a generic configuration.

Acknowledgment: This project has received funding from the European Union's Horizon 2020 research and innovation programme under the Marie Sklodowska Curie grant agreement No 813424.

References

[1] M. GHERGU, I. R. IONESCU: Structure-soil-structure coupling in seismic excitation and "city effect". International Journal of Engineering Science 2009, **47**(3):342-354.

[2] ANTONIO PALERMO, SEBASTIAN KRÖDEL, ALESSANDRO MARZANI, CHIARA DARAIO: Engineered metabarrier as shield from seismic surface waves. Scientific reports 2016, **6**(1):1-10.

[3] XINGBO PU, ANTONIO PALERMO, ZHIBAO CHENG, ZHIFEI SHI, ALESSANDRO MARZANI: Seismic metasurfaces on porous layered media: Surface resonators and fluid-solid interaction effects on the propagation of Rayleigh waves. International Journal of Engineering Science 2020, **154**:103347.

[4] H. LAMB: I. on the propagation of tremors over the surface of an elastic solid. Philosophical Transactions of the Royal Society of London. Series A, Containing papers of a mathematical or physical character 1904, **203** (359-371):1-42.

[5] XINGBO PU, ANTONIO PALERMO, ALESSANDRO MARZANI: Lamb's problem for a half-space coupled to a generic distribution of oscillators at the surface. arXiv preprint arXiv:2101.09997, 2021.