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# Combined Internal Resonances of Slacked Micromachined Resonators

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**Abstract:** The dynamics of micro/nanoelectromechanical systems (M/NEMS) curved beams have been thoroughly investigated in the literature, commonly for curved arch beams actuated with electrodes facing their concave surface. Except for few works on slacked carbon nanotubes, the literature lacks a deep understanding of the dynamics of slacked curved resonators, where the electrode is placed in front of the convex beam surface. In this paper, we investigate the dynamics of slacked curved resonators as experiencing internal resonance. The curved slacked resonator is excited using an antisymmetric partial electrode to activate both modes of vibration: symmetric and antisymmetric. The axial load is tuned to monitor the ratios between the natural frequencies of different modes of vibration. We explored the dynamics using Galerkin and multiple time scales (MTS) methods. The results indicate simultaneous 2:1 and 1:1 internal resonances between the second symmetric mode with the first symmetric and antisymmetric modes, triggering a variety of rich and complex dynamical behaviours.

**Keywords:** Slack curved beam, Internal resonance, Combined resonances, M/NEMS

## 1. Introduction

Energy transfer via internal resonance among various modes of vibration of M/NEMS has been exploited for many potential applications in recent decades [1], [2]. The inherent nonlinear nature and the low damping of these moveable structures present an ideal platform for activating internal resonances. Hence, understanding the nature of this phenomenon is crucial for their successful implementations. Curved beams, particularly, have been studied due to their cubic and quadratic nonlinearities that enable the activation of different types of internal resonances, such as 1:1, 2:1, and 3:1. Commonly, a MEMS arch resonator is actuated using an electrode facing its concave surface to induce snap-through motion. However, this configuration leads to higher actuation voltages. Despite the extensive research on different types of internal resonances in these MEMS resonators [3]–[5], there is a lack of characterizing them at a slack position, leading to lower actuation voltages and low power consumption. Here, we aim to study the combined 1:1 and 2:1 internal resonances of an initially curved beam, where the electrode is placed in front of the convex beam surface, using both Galerkin and MTS methods. The MEMS arch beam is axially tuned and electrostatically driven. Also, to enhance the activation of different modes, we actuate the arch beam electrostatically using a partial electrode configuration [3].

## 2. Results and Discussion

The axial load is chosen to reach a ratio 1:1 between the first symmetric and antisymmetric modes (at the crossing) and a ratio 2:1 with the second symmetric mode [3]. This makes the dynamic response more complex combining 1:1 and 2:1 internal resonances. The analytical study is based on a nonlinear Euler-Bernoulli shallow arch beam model. First, we solve the dynamic problem for different exci-

tation voltages using a multi-mode Galerkin method. Next, we expand the arch equation, accounting for the influence of different terms of quadratic and cubic nonlinearities, using the method of multiple time scales (by attacking the partial differential equation of motion directly) considering the combination of 1:1 and 2:1 internal resonances. As shown in Fig.1, the theoretical results, using both methods, show the nonlinear interaction between all contributing modes. The presence of different peaks and the phase portraits suggest the contribution of the second symmetric mode into the response via 2:1 internal resonance. The motion of the node point, the midpoint, around the first antisymmetric mode, suggests the contribution of the second symmetric mode to the response. The results prove the rich dynamic and nonlinear energy transfer of the slacked arch beam similar to the typical actuated MEMS arch resonators and with lower excitation voltages.

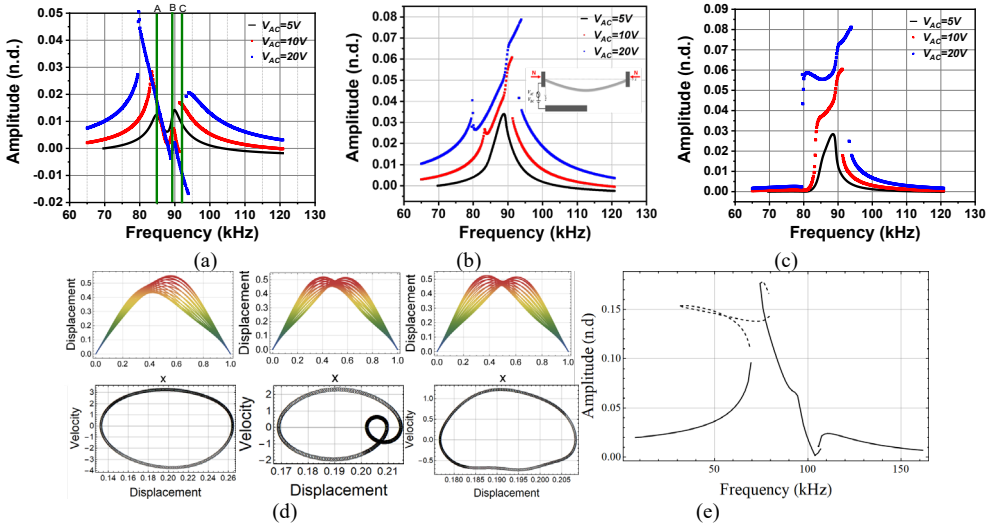


**Fig. 1.** Galerkin Results at (a) x=0.5, (b) x=0.25, and (c) x=0.75 (x denotes the normalized beam position with respect to the beam length). (d) Oscillations of the beam and phase portraits at different sections shown in (a) (A, B, and C from left to right). (e) MTS results of the dynamic response of the arch beam at $V_{DC}$=50V and $V_{AC}$ =10V. The inset of (b) presents a schematic of the arch beam.

## 3. Concluding Remarks

We investigated the combined 1:1 and 2:1 internal resonances among the first symmetric and antisymmetric and the second symmetric modes of an axially tuned and electrostatically actuated slacked arch MEMS resonator. A complex and rich dynamic behaviour was demonstrated. This work motivates further research to exploit internal resonances of slacked arch resonators for practical applications, such as sensors and frequency stability, thanks to the low actuation voltages.

## References


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