DSTA 2021
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16th INTERNATIONAL CONFERENCE
Dynamical Systems – Theory and Applications
December 6-9, 2021, On-line

Piecewise Linear Dynamics of a Cracked Beam with Hysteretic Damping

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Abstract: This study is concerned with the piecewise linear (PWL) dynamics of a cracked Euler-Bernoulli (E-B) beam with hysteretic damping. The mode 1 crack in the beam is modelled as a PWL spring at the crack location resulting in slope discontinuity during the crack opening. On crack closure, the frictional contact between the surfaces leads to forces that exhibit hysteresis and an empirical hysteretic damping model is incorporated. A semi-analytical approach in evolved and we present some of the interesting results emerging in the forced dynamics

Keywords: Nonlinear vibration, cracked beam, hysteretic damping, piecewise linear oscillator

1. Introduction

The presence of the crack in a structure affects its stiffness and dynamical characteristics. The opening/closing of the cracks leads to disparity in the interfacial stiffness, which is essentially nonlinear. The dynamical study of such structures is essential in engineering applications. Yokohama et al. [1] considered a modified line-spring model for a uniform E-B beam to study vibration characteristics. Abraham et al. [2] considered Timoshenko beam with a transverse crack to be of piecewise nature in the time domain incorporating dry friction at the crack interface. Chati et al. [3] considered bilinear frequency of the PWL model of the cracked beam and a PWL 2-DOF reduced model to study their dynamics. In addition to stiffness disparity, there is dissipation which can be nonlinear. Models based on the underlying dissipative mechanics are complicated, whereas the simpler low-dimensional models are empirical. Maiti et al. [4] studied the response of beams with internal dissipation modelled as the net averaged effect of a large number of randomly dispersed frictional microcracks incorporating hysteresis model.

In this study we consider PWL spring at the location of the crack owing to a change in stiffness due to opening/closing of the crack. The energy dissipation during the crack closure is considered as hysteretic damping and is invoked in a piecewise form. General forcing is considered to study the effect of crack location, depth and the damping on the dynamics. This study considers semi-analytical approach, method of averaging and Galerkin's method.

2. Mathematical Model

Consider a E-B beam with a crack modelled as two beams connected by a bilinear torsional spring at the crack location. The nondimensional equation of motion is

$$v_{yyyy} + v_{\tau\tau} + q_{b,y}(y,\tau) = f(y,\tau) \qquad (1a)$$

Where $0 \le y \le 1$ and the hysteretic damping is defined in the form

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$$q_b(y,\tau) = \begin{cases} \chi\theta(e,\tau)v_{yy}(y,\tau)\delta(y-e), v_{yy}(e,\tau) \geq 0 \\ 0, v_{yy}(e,\tau) < 0 \end{cases}$$
$$\dot{\theta}(e,\tau) = \kappa\{\theta_a + \beta \operatorname{sgn}(v_{yy}\dot{v}_{yy}) - \theta(e,\tau)\} |\dot{v}_{yy}|/(|v_{yy}| + \varepsilon) \quad (1b)$$

Where $v(y,\tau)$ is the transverse displacement, e is the crack location, χ is the damping parameter, θ is an internal variable and $\kappa, \theta_a, \beta, \varepsilon$ are constants governing the hysteretic damping behaviour [4]. The boundary conditions are, $v(0,\tau) = 0$; $v_y(0,\tau) = 0$; $v_{yy}(1,\tau) = 0$; $v_{yyy}(1,\tau) = 0$; $v(e_-,\tau) = v(e_+,\tau)$; $v_{yy}(e_-,\tau) = v_{yy}(e_+,\tau)$; $v_{yyy}(e_-,\tau) = v_{yyy}(e_+,\tau)$. The slope discontinuity is of the form,

$$\frac{v_y(e_+,\tau) - v_y(e_-,\tau)}{v_{yy}(e,\tau)} = \begin{cases} \alpha_1, v_{yy}(e,\tau) \geq 0 \\ \alpha_2, v_{yy}(e,\tau) < 0 \end{cases} \quad (2)$$

$v_{yy}(e,\tau) \geq 0$ implies crack closure and $\alpha_1 = 0$ resulting in slope continuity at $y = e$ and hysteretic damping is active (ref. $(1b)$). Whereas, $v_{yy}(e,\tau) < 0$ implies crack opening leading to slope discontinuity at $y = e$ and hysteretic damping is inactive (ref. $(1b)$). In the absence of dissipation, the equations of motion are solved in two regimes and the solutions are matched at the time instants when $v_{yy}(e,\tau) = 0$. The normal mode solutions form the basis for the Galerkin's method wherein the dissipation is introduced.

3. Results and Concluding Remarks

Undamped forced response for $f(y,\tau) = f_0\delta(y-1)\sin(\Omega\tau)$ is shown in Fig. 1 for varying Ω. For $\Omega = \omega_1(\alpha_{1,2} = 0)$, the first mode ($i = 1$) of the crack closed configuration resonates, but it goes off resonance when $v_{yy}(e,\tau) < 0$ resulting in modulated response (Fig. 1a). However, when the excitation frequency is equal to the bilinear frequency $\omega_{i(b)} = 2\omega_i(\alpha_{1,2} = 0)\omega_i(\alpha_{1,2} = 0.75)/\{\omega_i(\alpha_{1,2} = 0) + \omega_i(\alpha_{1,2} = 0.75)\}$, one can observe that the response grows unbounded (Fig. 1b). This is for the first mode ($i = 1$) and a more detailed study incorporating the effect of hysteretic damping, prediction of modulation envelope for higher modes ($i > 1$) will be included in the full version paper.

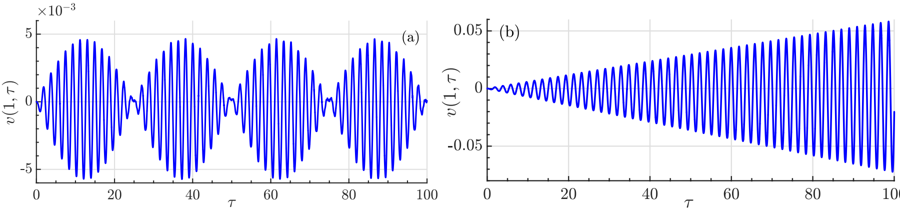

Fig. 1. Response $v(1,\tau)$ for (a) $\Omega = \omega_1(\alpha_{1,2} = 0)$, (b) $\Omega = \omega_{1(b)}$ for $f = 10^{-3}, \alpha_1 = 0, \alpha_2 = 0.75, e = 0.5$

Acknowledgment: VT acknowledges the assistantship provided by Ministry of HRD, India

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