DSTA 2021 dys-ta.com
16th INTERNATIONAL CONFERENCE
Dynamical Systems – Theory and Applications
December 6-9, 2021, On-line

Analytical investigation of a mechanical system containing a spherical pendulum and a fractional damper

JAN FREUDLICH[1*], DANUTA SADO[2]

1. Warsaw University of Technology, Faculty of Automotive and Construction Machinery Engineering, Warsaw, Poland [ORCID 0000-0002-0398-3271]
2. Warsaw University of Technology, Faculty of Automotive and Construction Machinery Engineering, Warsaw, Poland

* Presenting Author

Abstract: The presented work deals with a three degree of freedom system with a spherical pendulum and a damper of the fractional type. The system consists of a block suspended from a linear spring and a fractional damper, and a spherical pendulum suspended from the block. The viscoelastic properties of the damper are described using the Riemann-Liouville fractional derivative. The fractional derivative of an order of $0 < \alpha \leq 1$ is assumed. The nonlinear behaviour of the system in the vicinity of the internal and external resonances is studied. The multiple scales method has been used to obtain an approximate analytical solution to the studied system. The impact of a fractional order derivative on the dynamical behaviour of the system has been studied.

Keywords: spherical pendulum, fractional damping, nonlinear vibrations, multiple scale method,

1. Introduction

The presented work deals with a three degree of freedom system with a spherical pendulum and a damper of the fractional type. This work is a continuation of the authors' previous work [1]. The effect of fractional damping on the dynamic properties of a coupled mechanical system with a spherical pendulum is examined. It is assumed that the spherical pendulum is suspended to the block mass, which is excited harmonically in the vertical direction (Fig. 1). The oscillator contains a linear spring and a damper of a fractional type. The body of mass m_1 is subjected to the harmonic vertical excitation $F(t) = P\cos\nu t$.

The investigated system (Fig. 1) can be described by dimensionless equations of motion [1]. For small oscillations, after transformations the equations of motion can be written down in the form

$$\ddot{z} - a\ddot{\theta}\left(1-\frac{\phi^2}{2}\right)\left(\theta-\frac{\theta^3}{6}\right) - a\ddot{\phi}\left(\phi-\frac{\phi^3}{6}\right)\left(1-\frac{\theta^2}{2}\right) = p\cos(\mu_1\tau) + a\left[\dot{\phi}^2\left(1-\frac{\phi^2}{2}\right)\left(1-\frac{\theta^2}{2}\right) - 2\dot{\phi}\dot{\theta}\left(\phi-\frac{\phi^3}{6}\right)\left(\theta-\frac{\theta^3}{6}\right) - \dot{\theta}^2\left(1-\frac{\phi^2}{2}\right)\left(1-\frac{\theta^2}{2}\right)\right] - \gamma\dot{z}^{(\alpha)} - z \tag{1}$$

$$-\ddot{z}\left(1-\frac{\phi^2}{2}\right)\left(\theta-\frac{\theta^3}{6}\right) + \ddot{\theta}\left(1-\frac{\phi^2}{2}\right)^2 = 2\dot{\theta}\dot{\phi}\left(1-\frac{\phi^2}{2}\right)\left(\phi-\frac{\phi^3}{6}\right) - \beta^2\left(1-\frac{\phi^2}{2}\right)\left(\theta-\frac{\theta^3}{6}\right) \tag{2}$$

$$-\ddot{z}\left(\phi-\frac{\phi^3}{6}\right)\left(1-\frac{\theta^2}{2}\right) + \ddot{\phi} = -\dot{\theta}^2\left(1-\frac{\phi^2}{2}\right)\left(\phi-\frac{\phi^3}{6}\right) - \beta^2\left(\phi-\frac{\phi^3}{6}\right)\left(1-\frac{\theta^2}{2}\right) \tag{3}$$

where $\dot{f}^{(\alpha)}(\cdot)$ is the Riemann-Liouville fractional derivative defined as [2]

$$\dot{f}^{(\alpha)}(t) \equiv \frac{\mathrm{d}^{\alpha}}{\mathrm{d}t^{\alpha}} f(t) \equiv \frac{1}{\Gamma(1-\alpha)} \frac{d}{dt} \int_0^t \frac{f(\tau)\mathrm{d}\tau}{(t-\tau)^{\alpha}}, \ 0 < \alpha \leq 1 \tag{4}$$

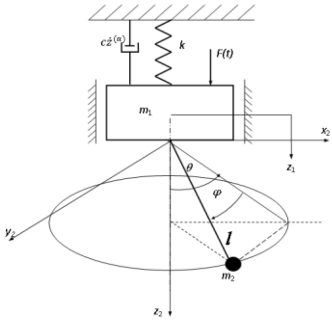

Fig. 1. Schematic diagram of the system [1]

The multiple scales method is used to find the approximate solution to the equations of motion of the system (1-3). We introduce independent variables $\{T_0, T_1, T_2 \dots\} = \{\tau, \varepsilon\tau, \varepsilon^2\tau \dots\}$ and parameters $\gamma = \varepsilon\tilde{\gamma}$, $p = \varepsilon^2\tilde{p}$. The solution can be represented by

$$z(t) = \varepsilon z_1 + \varepsilon^2 z_2 + \varepsilon^3 z_3 + \cdots, \ \theta(t) = \varepsilon\theta_1 + \varepsilon^2\theta_2 + \varepsilon^3\theta_3 + \cdots, \phi(t) = \varepsilon\phi_1 + \varepsilon^2\phi_2 + \varepsilon^3\phi_3 + \cdots \tag{5}$$

Substituting the equations (5) into equations (1-3) we obtain

For ε^1

$$D_0^2(z_1) + z_1 = 0, \ D_0^2(\theta_1) + \beta^2\theta_1 = 0, \ D_0^2(\phi_1) + \beta^2\phi_1 = 0 \tag{6}$$

For ε^2

$$D_0^2(z_2) + z_2 = -2D_0D_1(z_1) + aD_0^2(\theta_1)\theta_1 + \ aD_0^2(\phi_1)\phi_1 + \tilde{p}cos(\mu_1\tau) - \tilde{\gamma}D_0^{\alpha}(z_1) + a\left(\left(D_0^2(\phi_1)\right)^2 - \left(D_0^2(\theta_1)\right)^2\right)$$
$$D_0^2(\theta_2) + \beta^2\theta_2 = -2D_0D_1(\theta_1) + D_0^2(z_1)\theta_1, \ \ D_0^2(\phi_2) + \beta^2\phi_2 = -2D_0D_1(\phi_1) + D_0^2(z_1)\phi_1 \tag{7}$$

Next, substituting solution to the equations (5) into equations (6) and eliminating terms that produce secular terms we obtain conditions $2\beta = 1, \ \mu_1 = 1, \ 1 - \beta = \beta$. Next, we introduce $1 - \beta = \beta + \varepsilon\sigma_2, \ \mu_1 = 1 + \varepsilon\sigma_1, \ \varepsilon T_0 = T_1$, and we investigate a steady-state solution.

3. Concluding Remarks

The impact of the order of the fractional derivative on small nonlinear oscillation of the system containing a spherical pendulum has been studied. The performed analysis show that use of the fractional damping has an impact on the small oscillation of the system.

References

[1] FREUNDLICH J, SADO D: Dynamics of a coupled mechanical system containing a spherical pendulum and a fractional damper. *Meccanica* 2020, **55**:2541-2553.

[2] PODLUBNY I: *Fractional Differential Equations*. Academic Press: San Diego, 1999.