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On the stability of a slip controlled two-axle vehicle with multiple time-delays

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Abstract: In the present work, stability analysis of a two-wheeled vehicle is performed, with slip control applied to the wheels. The model contains two wheels equipped with dynamic brush tyre models and two digital PID torque controllers that affect the wheels through the brake system. Feedback delay of the control loops are considered; thus the mathematical model of the system contains delay differential equations. The stability charts are constructed, and bifurcations are analysed. Finally, simulation results of the nonlinear model is presented.

Keywords: time-delay, stability analysis, vehicle dynamics

1. Introduction

Nowadays, vehicle safety is one of the leading topics in industrial and scientific research. There are essential computer controller subsystems in our ground vehicles such as ABS or ESP, and the number of safety relevant vehicle dynamics control systems are growing year over year. Therefore, detailed modelling of the vehicle system is more and more important. In the presented work, stability analysis of a two wheeled vehicle is performed.

2. Models and Methods

Firstly, the mechanical model of the vehicle is introduced. The model consists of two wheels that are connected with a rigid rod. The wheels can be separated into two parts, a rolling rigid disk, and a tyre model that can describe the connection between the wheel and the road surface. Dynamical brush type tyre model is used in the study, which is a continuum model containing small elastic bristle elements [1]. The equations of motion of the vehicle system can be read as

$$m\dot{V}(t) = F_{XF}(u_F(x,t)) + F_{XR}(u_R(x,t)), \tag{1}$$

$$J_{\{F,R\}}\dot{\Omega}_{\{F,R\}}(t) = T_{D\{F,R\}} - T_{B\{F,R\}}(t) - RF_{X\{F,R\}}(u_{\{F,R\}}(x,t)), \tag{2}$$

$$\dot{u}_{\{F,R\}}(x,t) = R\Omega_{\{\mathrm{F,R}\}}(\mathrm{t}) - V(t) + u'(x,t)R\Omega_{\{\mathrm{F,R}\}}(t). \tag{3}$$

Here, V and $\Omega_{\{\mathrm{F,R}\}}$ denote the longitudinal velocity of the vehicle and the angular velocity of the front and the rear wheels, respectively. $J_{\{F,R\}}, m$ and $R_{\{F,R\}}$ are for the inertia of the wheels, the gross weight of the vehicle, and the dynamic radius of the wheels, respectively. The deformation function of the bristle elements of the tyre is $u(x,t)$, where x is the spatial coordinate of the contact segment between the tyre and the road surface. The force $F_{X\{F,R\}}$, that is generated by the contact, depends on the deformation function $u_{\{F,R\}}$. Equations (1) and (2) are ordinary differential equations, while (3) is a partial differential equation.

The driveline acts the wheel with driving torque T_D, while the brake system exerts with braking torque T_B. The brake system is in the focus of the present study. In the investigated scenario, the brake controllers actuate the brake system in order to influence the state of the wheels. There are two separate controllers in the model, one for the front, and one for the rear axle's wheel. Thus, the model contains two feedback loops, which are able to influence each other in a mechanical way through the rigid connection between the two wheels.

Last but not least, there are two important effects that should be considered in our model, first is the time-delay in the feedback loop, and the other is the effect of sampling. The formula of the delayed control signal can be written as

$$T_{B\{F,R\}}\big(t,\tau_{\{F,R\}}\big) = k_P e\big(t-\tau_{\{F,R\}}\big) + k_I \int_0^{t-\tau_{\{F,R\}}} e(T)dT + k_D \dot{e}\big(t-\tau_{\{F,R\}}\big), \tag{4}$$

where k_P, k_I, k_D are the controller gains, time-delay is denoted by τ, and e is the error signal. Time-delay and sampling are connected. Considering only constant time-delay, delay differential equations arise [2]. If sampling is taken into account, the delay transforms to a time-varying delay, and the mathematical model of the system contains periodic DDEs [3]. It is important to note, that different time-delays can be assumed for the two separate controllers.

Finally, the full delayed feedback loop with the mechanical model of the vehicle is treated in the research. After linearization, semi-discretization is applied, which we used to construct the mathematical model. With this approach, the continuous time mechanical model, and the discrete time controller can be handled, and stability analysis can be performed by calculating the characteristic multipliers of the resulting discrete difference equation which governs the full system.

Stability maps, and effect of changing of different system parameters on the stability maps are presented. As the system is of neutral type, if constant time delay is considered, boundaries can be found on the stability map, where multiple Hopf-bifurcations can occur. It is pointed out in the study, that in the sampled system, the number of these bifurcations is in connection with the delay. Finally, the results are compared with simulations of the nonlinear model.

Performance characteristics are investigated as well. Using simulations and a genetic algorithm, different optimizations are performed for various parameters.

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