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Coupled System of Stochastic Neural Networks with Impulses, Markovian Switching, and node and connection Delays

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Abstract: In this paper we give sufficient conditions for p-th moment general decay stability of a model of coupled stochastic neural networks, which apart from Markovian switching, impulses and node delays, also includes interconnection delays. The model is more general than the results in the literature and thus the known methods for stability analysis can not be directly applied. We give the main result using the M-matrix theory and additionally, we analyse stability with respect to a general decay function, which includes the logarithmic, exponential and polynomial function as a special case. To the best of our knowledge, the results are original and have not been published yet.

Keywords: coupled systems, stochastic neural networks, p-th moment stability, general decay function

1. Introduction

Coupled system of neural networks are of special interest in the analyses of dynamical systems, since their dynamics depends on the individual dynamics of the included networks, as well as on the way in which these networks are interconnected. Different aspects of coupled networks are studied, with an emphasis on synchronization and stability. Many results have been reported in the literature and here we give only few of them which study the stability of coupled systems on networks, both deterministic and stochastic [1, 2, 3, 4].

In this paper we are interested in the stability analyses of a model of coupled stochastic neural networks (CSNN) which includes node delays, impulses, Markovian switching and interconnection delays. There are different results on coupled systems of networks in the but to the best of our knowledge this model is not considered and the presented results are original.

2. Results and Discussion

The model we consider is given by a system of stochastic differential equations. The dynamics of the i-th neuron in the k-th vertex of the CSNN, $i, k \in N$ at any moment $t \geq t_0$, $t \neq t_m$ is given by

$$
\begin{aligned}
dX_i^{(k)}(t) &= -h_i^{(k)}\left(X_i^{(k)}(t), r(t)\right)\Bigg[c_i^{(k)}\left(t, X_i^{(k)}(t), r(t)\right) \\
&- \sum_{j=1}^{n} a_{ij}^{(k)}(t, r(t)) f_j^{(k)}\left(X_j^{(k)}(t), r(t)\right) - \sum_{j=1}^{n} b_{ij}^{(k)}(t, r(t)) g_j^{(k)}\left(X_{j,\tau_k}^{(k)}, r(t)\right) \\
&- \sum_{j=1}^{n} d_{ij}^{(k)}(t, r(t)) \int_{-\infty}^{t} l_{ij}^{(k)}(t-s) k_j^{(k)}\left(X_j^{(k)}(s), r(s)\right) ds\Bigg] dt \\
&+ \sum_{j=1}^{n} \eta_i^{(kj)}\left(t, X_i^{(k)}(t), X^{(j)}(t), X_{\tau_{kj}}^{(j)}, r(t)\right) dt + \sum_{j=1}^{n} \sigma_{ij}^{(k)}\left(t, X_j^{(k)}(t), X_{j,\tau_k}^{(k)}, r(t)\right) dW_j(t) \\
&+ \sum_{j=1}^{n} \varsigma_i^{(kj)}\left(t, X_i^{(k)}(t), X^{(j)}(t), X_{\tau_{kj}}^{(j)}, r(t)\right) dW_j(t),
\end{aligned}
$$

and for $t = t_m$, $m \in \mathbb{N}$,

$$
X_i^{(k)}(t) = \mathcal{I}_{im}\left(X_1^{(k)}(t^-), \ldots, X_n^{(k)}(t^-)\right) + \mathcal{J}_{im}\left(X_1^{(k)}(t^- - \tau_k(t^-)), \ldots, X_m^{(k)}(t^- - \tau_k(t^-))\right), \quad (4.47)
$$
$$
X_i^{(k)}(t_0 + s) = \xi_i^{(k)}(s), \; s \in (-\infty, t_0],
$$

This model is generalization of the one considered in [DSTA 2019], where stability analysis was discussed using global Lyapunov function, Razumikhin method and graph theoretical approach. These methods are not suitable for the analysis of our model, due to the presence of interconnection delays. We give suitable assumptions for the components of the network and extend some known results. We state the main result of our paper for p-th moment stability with respect to a general decay function and prove the results using M-matrix theory.

3. Concluding Remarks

In this paper we discuss the p-th moment stability of coupled stochastic neural network which generalizes many models in the literature and even more, we consider a generalised decay function. The results are based on M-matrix theory and to the best of our knowledge, they have not been published yet.

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