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Nonlinear Dynamics of Atomic Systems in a Free State and in an External Electromagnetic Field: Chaos and Attractors

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Abstract: The fundamentals of a novel mathematical approach to studying deterministic chaos and strange attractors in dynamics of nonlinear processes in atomic and molecular systems in an electromagnetic field. The new quantum-dynamic models (based on the finite-difference solution of the Schrödinger equation, optimized operator perturbation theory and realistic model potential for quantum systems) and advanced nonlinear analysis and a chaos theory methods (power spectrum analysis, spectral algorithms, the correlation integral algorithm, the fractal method, the Lyapunov's exponents and Kolmogorov entropy analysis etc) are realized in order to provide a correct treatment a chaotic dynamics of atomic systems (the features of spectra and field provided chaotic ones). Availability of multiple resonances (autoionization or field provided stark Zeeman type ones) with super little widths in spectrum of an atomic system in external field is treated and provided by interference phenomena and fluctuations. Dynamics of resonances in atomic spectra is studied and the topological and dynamical invariants are calculated.

Keywords: dynamics of atomic systems, spectral features, chaos and attractors

1. Introduction. Quantum-Dynamic and Chaos-Geometric Approaches to Atomic System Dynamics

An analysis of the chaotic phenomena in quantum systems was carried out not only based on the methods of classical mechanics (in fact, within the framework of the Newtonian dynamics), but also on the basis of semiclassical or semi-quantum methods, in particular, the method of quantum trajectories (quantization of classical mechanics), and path integrals by Feynman-Higgs, the Gutswiller's theory of "periodic orbits", the Delos closed orbit method, complex coordinate method, a random matrix theory, diagonalization methods and some others (e.g. [1,2]). New field of investigations of chaotic effects in theory of quantum systems has been provided by a great progress in a development of a chaos and dynamical systems theory methods. In this work we present a novel mathematical approach to studying deterministic chaos and strange attractors in dynamics of nonlinear processes in atomic and molecular systems in an electromagnetic field. The total scheme for studying chaos-dynamical phenomena in quantum systems (in particular, atomic systems in magnetic, crossed electric and magnetic fields, Rydberg atoms in a electromagnetic field, molecular systems in a infrared electromagnetic field etc) and computing the topological and dynamical invariants in application to quantum systems include the following [1-4]:

A) Quantum-dynamical computing of quantum systems: Schrödinger (Dirac) equation for quantum system in an external field (numerical solving, the finite differences , model potential , operator per-

turbation theory etc methods); Preliminary analysis and processing dynamical variable series of physical system;
B) Preliminary study and assessment of the presence of chaos: the Gottwald-Melbourne test; Fourier decompositions, irregular nature of change – chaos; Spectral analysis, Energy spectra statistics, the Wigner distribution, the spectrum of power, "Spectral rigidity";
C) The multi-fractal geometry: computation time delay τ using autocorrelation function or mutual information; Determining embedding dimension by the method of correlation dimension or algorithm of false nearest neighbouring points; Calculation of multi-fractal spectra; wavelet analysis;
D) Computing global Lyapynov's exponents, Kaplan-York dimension, Kolmogorov entropy, average predictability measure; Methods of nonlinear prediction (classical and quantum neural network algorithms, the algorithm optimized trajectories, stochastic propagators, memory functions etc...;

The key idea in the study of the spectra of chaotic systems and, in particular, quantum systems, is provided by the fact that a definition of quantum chaos is interpreted primarily as a property of a group of states of the spectra of the system. It is the interpretation of one of the mechanisms of quantum chaos through the induction of resonances in the spectrum of the system, their strong interaction with subsequent overlapping, the emergence of stochastic layers and further transition to a global stochasticity in the system.

2. Results and Concluding Remarks

Firstly, the results of the modelling a chaotic dynamics for the Rydberg alkali (Li, Rb, Cs, Fr in states with the principal quantum number n~31-100, m= 0,1) atoms in a static magnetic (B = 4.5T) and oscillating electric field with frequency ω=102Мгц (ε=-0.03,ϖ=0.32, γ-1/3 in the range 35-50; f = 0.000-0.070) are presented. Secondly, there are presented data on the resonances of Rydberg Li in the DC electric field F = 2.1-2.5kV/cm, Rb in the DC F = 2.189- 6,416 kV/cm, n = 18-23 and chaotic ionization dynamics of Rydberg Li, Rb, Yb (Li: n = 41-70; Rb: n = 51-70; Yb: n = 60-80) in a microwave field (F = (1.2-3.2) 10^{-9} a.u.; f= 8.87GHz, 36GHz) with in a good agreement with available experimental data. Thirdly, for the first time to solve a class of problems related to the search for the phenomenon of quantum chaos in the spectra the new combined quantum and chaos-geometric approach (including analysis of level statistics and a group of new or improved methods of chaos theory) applied to an analysis of the spectra of heavy atomic systems (Yb, Tm, U). It is shown that the distribution of the parameter "nearest - level spacings "is close to the Vigner distribution. It has given a consistent theoretical interpretation to a phenomenon of a quantum chaos and described a strong nonlinear interaction of resonances with appearance of spectral stochastic layers with fusion.

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