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Dynamic Integrity of Hyperelastic Spherical Membranes

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Abstract: There are several applications of thin-walled spherical elastomeric membranes in civil and aerospace engineering, bioengineering and biology. Both geometrical and material nonlinearities play an important role in their static and dynamic behavior. Due to their high nonlinearity they present under pressure load multiplicity of stable solutions, which affect their dynamic integrity due to the competing basins of attraction. Also their response depends on the chosen constitutive law. Here the Ogden model is adopted, due to its generality, and the nonlinear equations of motion are obtained for a preloaded membrane. This work investigates the dynamic integrity of a pressure loaded spherical membrane considering global (GIM) and local (LIM) integrity measures and the integrity factor (IF). For this, initial conditions are sampled in the phase space based on the Monte Carlo method. The numerical results demonstrate the influence of competing solutions on the global dynamic behavior and safety of the structure.

Keywords: spherical membrane, Ogden constitutive model, integrity measures, Monte Carlo method

1. Introduction

Thin-walled spherical elastomeric membranes can undergo large elastic deformations. However, the nonlinear behaviour under high static pressure or large amplitude vibrations depends on the hyperelastic material modelling [1]. Furthermore, multiple solutions can coexist, resulting in competing basins of attraction with varying topologies [1], which can be quantified by different integrity measures [2]. In this work, the numerical procedures proposed in [2] are used to estimate the global (GIM) and local (LIM) integrity measures and the integrity factor (IF) of a pressure loaded spherical membrane.

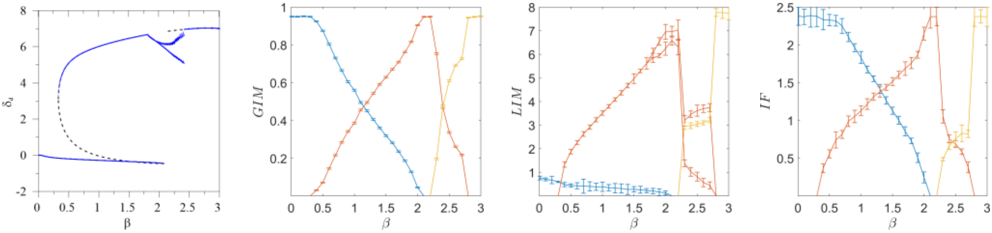
2. Results and Discussion

A closed homogeneous, isotropic, incompressible and hyperelastic spherical membrane with thickness assumed much smaller than the initial radius is considered. The deformed sphere can be described by the theory of membranes under finite deformations. Only the first vibration mode, consisting of the membrane inflation and deflation (breathing mode), is addressed, allowing a complete static and dynamic description in terms of the radial stretch δ [1]. The equation of motion is extremely dependent on the constitutive model. Considering an Ogden material, due to its generality, it takes the nondimensional form

$$\ddot{\delta}_d + \left(2\omega_0\zeta + \nu\delta_d^2\right)\dot{\delta}_d + \sum_{i=1}^{n}\frac{\mu_i}{\mu_1}\left[\left(\delta+\delta_d\right)^{\alpha_i-1} - \left(\delta+\delta_d\right)^{-2\alpha_i-1}\right] = Q_{sta}\left(1+\beta\cos\Omega t\right)\left(\delta+\delta_d\right)^2. \qquad (1)$$

where dots represent derivatives with respect to the nondimensional time $\tau = t\,(4\mu_1/\gamma a^2)^{1/2}$, ω_0 is the natural frequency, Q_{sta} is the static preload, β and Ω are the forcing magnitude and frequency of the radial harmonic excitation, ζ and ν are the linear and nonlinear damping coefficients. In addition, μ_i and α_i are the material parameters of Ogden model with number of terms n. For details regarding the model formulation, refer to Silva et al. [1].

Rich dynamics can be displayed by eq. (1), depending on the parameters' values, as pointed out in [1]. The pressure loaded membrane displays two potential wells. A typical bifurcation diagram as a function of the forcing magnitude β is shown in Fig. 1(a), displaying a stable branch that loses stability through a saddle-node bifurcation, where a dynamic jump to a large amplitude solution can occur. Another saddle-node bifurcation gives rise to a competing solution branch. The integrity measures are reported in Figs. 1(b)-(d), estimated through Monte Carlo algorithms [2] with a total of 10000 initial conditions sampled uniformly. Three solutions are identified, corresponding to the low amplitude oscillations (blue) and large amplitude oscillations (red and orange). The same trait is observed for all measures, with the blue and red basins being progressively eroded. The orange solution only appears after the blue is completely eroded. The error bars of the algorithm are also reported.



(a) (b) (c) (d)

Fig. 1. Integrity analysis of the hyperelastic spherical membrane. Bifurcation diagram (a), GIM (b), LIM (c), IF(d). ($n = 3$, Ogden OSS2 model in [1], $Q_{sta} = 0.15$, $\Omega = 2.1514$, $\omega_0 = 2.1514$ and $\zeta = \nu = 0.01$)

3. Concluding Remarks

In this work the dynamic integrity of a hyperelastic spherical membrane is evaluated via a Monte Carlo approach. This methodology can evaluate with precision the dynamic integrity measures GIM, LIM and IF without the actual basin calculation. The integrity analysis shows that the occurrence of coexisting stable pre- and post-buckling solutions leads to competing basins of attraction. Clarifying their erosion process through the integrity measures enables the engineer to evaluate the system safety in a dynamic environment.

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