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On the stability of sampled-data systems with dry friction

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Abstract: This paper focuses on the stabilization effect of dry friction in sample-data systems that use discrete-time state-feedback in position control applications. The results are presented through the example of a single-degree-of-freedom effective system model. This paper shows that in these systems, the destabilizing effect of sampling can still be compensated to some extent by the presence of dry friction resulted in an unstable limit cycle. The domain of attraction of the zero reference position as the fixed point is also presented in this paper.

Keywords: sampled-data nature, dry friction, concave envelope, negative viscous damping

1. Introduction

One of the fundamental tasks of mechatronics is position control. In these systems, the main aim of the applied controller is to drive the system into the desired position, which is typically achieved by state feedback. In the analysis of these systems, the effect of friction is often neglected due to the application of a suitable friction compensation algorithm [1] or because it results in a conservative stability condition. It is also a common practice to neglect the effect of sampling and quantization. Although these simplifications can be reasonable from the engineering point of view, explaining some intricate vibration phenomena requires the handling of more accurate system models. For example, the effect quantization may lead to chaotic motions even at high sampling rates [2], the effect of sampling can result in multi-frequency vibration even in the case of a single-degree-of-freedom system model [3], or the sampled-data systems can have special concave envelope vibrations when only dry friction stabilizes the motion [4].

2. Effective system model with dry friction

To illustrate the stabilization effect of dry friction, a single-degree-of-freedom mechanical system is considered where discrete-time state feedback with zero-order-hold signal recognition is used to drive the system into the zero reference position. The resulting governing equation of motion is

$$m\ddot{x}(t) + f_C \operatorname{sgn}(\dot{x}(t)) = -k_p x_j \quad \text{with} \quad t \in \left[t_j, t_j + \tau\right), \tag{1}$$

where $x(t)$ is the generalized coordinate as a function of time t, m is the generalized mass that takes its meaning based on the definition of x, and f_C denotes the magnitude of dry friction force. The control gain is k_p, and $t_j = j\tau$ with $j \in \mathbb{Z}$ is the jth sampling instant, where τ is the sampling time. In order to analyse the dynamic behaviour of the system, Eq. (1) can be solved for the discrete state variables $\mathbf{x}_j = [x_j \quad v_j]^{\mathrm{T}}$, where $x_j = x(t_j)$ and $v_j = \dot{x}(t_j)$ represents the sampled position and velocity at the beginning of the jth time interval, in the form of a non-homogeneous map $\mathbf{x}_{j+1} = \mathbf{A}\mathbf{x}_j - \mathbf{a}\operatorname{sgn}\left(\dot{x}(t)\right)$, which is valid between velocity reversals.

Effective continuous-time system model with friction

First, the case is investigated when the effect of dry friction is neglected. The corresponding results serve as a reference to examine the effect of friction. The dynamic behaviour of the frictionless system is represented by the roots $z_{1,2}$ of the characteristic equation of matrix **A**. When the parameter $p = k_p\tau^2/m$ is in the range $0 < p < 16$, then the characteristic roots are complex with non-zero imaginary part, i.e., $z_{1,2} = \rho\,\exp(\pm i\vartheta)$. It results that the magnitude ρ of $z_{1,2}$ is in the range $1 < \rho < 3$, which means that the frictionless system has unstable oscillations around the reference position. By neglecting the higher harmonics due to sampling, the motion can be characterized by a damped oscillator with negative viscous damping term to model the unstable behaviour. The resulted in the effective continuous-time model is

$$\ddot{x}(t) + f_0\omega_n^2\,\mathrm{sgn}(\dot{x}(t)) = -\omega_n^2 x(\mathrm{t}) + 2\zeta\omega_n\dot{x}(\mathrm{t}), \tag{2}$$

where $f_0 = f_C/k_p$, the undamped natural angular frequency ω_n, and the damping ratio ζ. For further details, the reader is referred to [3] and [4].

Stability in the presence of friction

First, the solution of the effective continuous-time model in Eq. (2) is needed. Assuming that the initial conditions are $x_0 > 0$ and $v_0 = 0$, the motion takes place with negative velocity. With these, the solution $x^-(t)$ can be determined until the first velocity reversal, which happens at $t = \pi/\omega_d$. If there is a periodic solution, the condition $x^-(\pi/\omega_d) = -x_0$ has to be satisfied resulted in the critical initial position as

$$x_0^* = f_0\,\coth\left(\frac{\zeta\omega_n}{\omega_d}\frac{\pi}{2}\right)\ \text{ with }\ \omega_d = \omega_n\sqrt{1-\zeta^2}. \tag{3}$$

In the case of non-zero initial velocity with arbitrary initial position, the solution of Eq. (2) is $x^+(t)$. Based on $x^+(t)$, the elapsed time for the first vibration peak can be determined as

$$t^\cap = \frac{1}{\omega_d}\,\mathrm{atan}\left(\frac{x_0 + f_0 + \zeta v_0}{v_0\sqrt{1-\zeta^2}}\right). \tag{4}$$

If the time $t^\cap$ is substituted back to $x^+(t)$, the first maximum position is $x^\cap = x^+(t^\cap)$. Finally, if the condition $x_0^* = x^\cap$ holds, there is also a periodic solution.

3. Concluding Remarks

In this paper, the main characteristics of sampled-data systems were investigated by considering dry friction as the primary source of physical dissipation. For the analysis of the sampled-data system with dry friction, an effective continuous-time model was derived. It was also shown when the effect of dry friction is also taken into account, the system can become sensitive to the initial conditions, and a limit cycle develops around the desired position.

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