DSTA 2021
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16th INTERNATIONAL CONFERENCE
Dynamical Systems – Theory and Applications
December 6-9, 2021, On-line

Nonlinear Dynamics of Multi-Stable Systems

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Abstract: Nonlinear dynamics of different types of multistable systems is presented in the paper. As an example a multistable shell which exhibits several equilibria in a static configuration and snap-through effect in its dynamic response is investigated. On the basis of the experimental tests, a one degree of freedom model in a form of quintic oscillator is proposed. The local and global dynamics of a bistable oscillator is investigated to detect reversible snap-through effect.

Keywords: nonlinear vibrations, multistable systems, snap-through, control, energy harvesting

1. Multi-Stable Systems

Multi-stability is one of the main futures of nonlinear systems. Duffing's oscillator may serve as a classical example where a zone with three different solutions may exist. Moreover, for certain excitation conditions a number of solutions in the Duffing model may increase up to five, as presented in [1]. However, the Duffing type equation has just one equilibrium position, which can be stable or unstable, depending on a shape of a potential function. In a classical Duffing equation the equilibrium is stable and the potential function has just one minimum.

Systems with multi equilibria belong to another group of the multi-stable structures [2]. This type of multi-stability can be created artificially by adding nonlinear external force, for example magnetic force produced by two magnets. The introduced nonlinear force enables to design a potential function which may have two potential wells. Another option is to add axial force to the structure which often may occur naturally due to increased temperature, for example. Then the system operating close to a buckling point has two potential wells, corresponding to two equilibria.

The composite technology offers new possibilities in creating multi-stable structures. The bistability, with associated snap-through mechanisms (a rapid jump from one to another equilibrium) is attractive for a design of efficient energy harvesters [3]. Most of the studies devoted to the dynamics of bistable laminates are characterized by symmetric stable configurations with free boundaries. A special design of a laminate shell with asymmetric configuration has been proposed in [4] where a few equilibria have been detected. The shell designed in [4] is investigated in this paper to detect local (in-well) and global dynamics with the snap-through effect.

2. Shell Model and Results

The studied shell is presented in Fig.2. A number of layers and the layout is designed to get shell multi-stability as presented in [4].

Fig. 2. Multistable shell I-state (a), C-state (b) and function of potential energy with indicated I and C states (c).

In Fig.2 two stable states are presented, I-state (Fig.2a) and C-state (Fig.2b). A corresponding function of potential energy, with two local minima indicated in Fig.2(c), confirm the bistability of the system. On the basis of experiment a reduced shell model is derived in a form of a quintic oscillator which represents dynamics of the shell observed experimentally. The model is used to predict in-well behaviour and the snap-through effect.

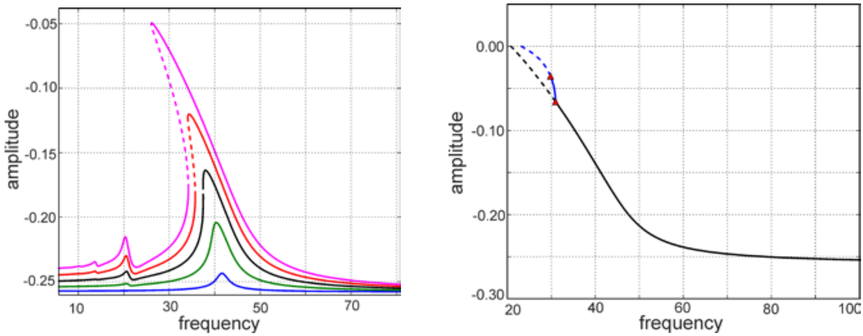

Fig. 2. Resonance curves of a multistable shell structure around I-state, (a) softening effect for selected amplitude of excitation, (b) period doubling indicating possible transition to snap-through effect

The numerical solutions of the elaborated model shows softening effect of the resonance curves for in-well dynamics, as presented in Fig.2(a) for I-state. For large amplitude of excitation the period doubling is observed (Fig.2b) which may lead to the snap-through with chaotic global dynamics.

Acknowledgment: The paper is partially supported from grant PO19MO15 and co-financed by the Polish National Agency for Academic Exchange PPN/BIL/2018/2/00076.

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