2021 DSTA dys-ta.com
16th INTERNATIONAL CONFERENCE
Dynamical Systems – Theory and Applications
December 6-9, 2021, On-line

About the Target-Attacker-Defender Optimal Problem,

CHERKASOV OLEG[1*], MAKIEVA ELINA[2], MALYKH EGOR[3]

1. Lomonosov Moscow State University, Moscow, Russia [0000-0003-1435-1541]
2. Lomonosov Moscow State University, Moscow, Russia [0000-0002-0134-3350]
3. Lomonosov Moscow State University, Moscow, Russia [0000-0002-5081-0584]

* Presenting Author

Abstract: The Target-Attacker-Defender problem is considered. Assumed that all participants move in a horizontal plane with velocities of constant modulus. The Attacker uses the pure chase method to pursue the Target. The Defender launched from the Target's wingman and the role of Defender is to minimize the distance to the Attacker when the Attacker approaches the Target at a given distance. The Defender's strategy is also a method of pure pursuit. The angular velocity of rotation of the Target velocity vector considered as a control. The structure of the dynamic system allows to reduce it to a system of less dimension. In the reduced system, the angle between velocity vector and line-of-sight Target-Attacker is considered as a new control variable. Pontryagin maximum principle procedure allows to reduce the optimal control problem a boundary-value problem (BVP) for a system of nonlinear differential equations of the fourth order. The system of the BVP consists from the initial variables and does not includes co-state variables. For solving the BVP, the shooting method is applied. The results of solving the BVP for various values of parameters demonstrated.

Keywords: Target-Attacker-Defender problem, Pure pursuit, Optimal control

1. Introduction

Traditionally, two approaches are used to investigate the Target-Attacker pursuit-evasion problem. The first one is based on the application of methods of the theory of differential games. The second approach assumes that the strategy of the pursuer is known and fixed, and the task is to build an optimal strategy for the evader [1]. One of the advantages of the second approach is the ability to use more realistic models of the dynamics of participants. In the pursuit-evasion problems for three objects, the Target-Attacker-Defender (TAD), in the differential game, both independent actions of all participants and cooperation between the target and the defender are possible [2]. TAD problem with three participants is also investigated in the case when the strategy of one of them is fixed [3]. In this paper, we consider the case when the strategy of both the Attacker and the Defender is fixed. As a prescribed strategy, the method of pure pursuit is adopted, in which the velocity vector of the pursuer is directed exactly along the line of sight of the pursuer-target. In this formulation, the trajectories of the other participants are determined by Target motion. Therefore, the problem is to find the optimal strategy of the Target. The goal function is the distance between the Defender and the Attacker, when the distance between the Attacker and the Target becomes equal to a given value. Thus, the duration of the process is free. It is assumed that all objects move in a horizontal plane with velocities of constant modulus.

2. Problem Formulation

Equations of motion are as follows:

$$\begin{cases} \dot{\varphi} = -\dfrac{\sin\varphi}{R} + u, \\ \dot{\theta} = -\dfrac{a}{R}\sin\varphi + \dfrac{1}{r}\sin\theta, \\ \dot{R} = a\cos\varphi - 1, \\ \dot{r} = -b - \cos\theta. \end{cases} \tag{1}$$

where φ is the angle between LOS Target-Attacker and the velocity vector of Target, θ is the angle between the velocity vector of Attacker and Defender, R – normalized distance between Attacker and Target, r – normalized distance between Defender and Attacker, u is the control variable, a is the ratio of Target velocity modulus to Attacker velocity modulus, b is the ratio of Defender velocity modulus to Attacker velocity modulus.

Boundary conditions are as follows:

$$\theta(0) = \theta_0, R(0) = R_0, r(0) = r_0, \varphi(0) \text{ is free}, R(T) = R_T \tag{2}$$

The goal function is:

$$J = r(T) \to \min_u \tag{3}$$

where T is free.

The problem (1) - (3) is Mayer optimal control problem.

References

[1] BEN-ASHER, J.Z., CLIFF, E.M: Optimal evasion against proportionally guided pursuer. *Journal of Guidance, Control and Dynamics.* 1989, **12**(4):598-600.

[2] TURETSKY, V., GLIZER V.,Y: Open-loop solution of a defender–attacker–target game: penalty function approach. *Journal of Control and Decision.* 2018 6(1):1-25.

[3] GARCIA E., CASBEER D.W., KHANH PHAM D., PACHTER, M: Cooperative aircraft defense from an attacking missile. *Proceedings of the IEEE Conference on Decision and Control*. 2015:2926-2931.