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Zermelo Navigation Problem with State Constraints,

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Abstract: The article uses the example of the Zermelo navigation problem to illustrate a simple way to address state constraints of a certain type. The problem of planning time-minimum trajectory of an autonomous aircraft operating in a steady, homogeneous flow field is considered. The simple particle model of the aircraft is considered. The particle moves in a horizontal plane with a constant modulus velocity relative to the flow of the medium. The angular velocity of rotation of the particle velocity vector is considered as the control variable. The angle between the velocity vector and the horizontal axis is subjected to a phase constraint. The structure of the dynamic system allows to reduce the optimal problem to the problem of a smaller dimension. In reduced problem the state constraints transform to the constraints on the control variables. For the reduced problem, the optimal synthesis is designed. Next, for the original problem, the sequence and the number of the arcs with motion along state constraints are determined. The control law in the initial problem is established.

Keywords: Zermelo navigation problem, state constraints, optimal synthesis

1. Introduction

Solving optimal control problems in the presence of phase constraints is a complex task. The fact that no constructive methods have been developed to solve such problems makes each problem solved valuable. Significant progress can be made if the structure of the optimal trajectory, the number of arcs moving along the constraints, and their sequence are known. In this article, using Zermelo navigation problem [1] as an example, we demonstrate an approach that allows us to construct an optimal synthesis for problems with phase constraints of a certain type. A fairly complete review of the solutions of the Zermelo problem for various types of flows and the rigorous construction of optimal synthesis can be found, for example, in [2]. Of particular interest is the specified problem in the presence of state constraints on the coordinates. In this case, as a rule, the solution could be found using numerical simulation based on the method of penalty functions [3]. The problem is greatly simplified if the structure and sequence of extreme arcs are known. The path-planning problem therefore reduces to identifying the switching points at which straight and trochoidal path segments join to form a feasible path and choosing the true minimum-time solution from the resulting set of candidate extremals [4]. Zermelo problem with state constraints, imposed to the coordinates of the point, considered in [5].

This article describes a simple way to construct an optimal path in the presence of phase constraints of a certain type.

2. Results and Discussion

Equations of motion have a form:

$$\begin{cases} \dot{x} = v\cos\theta + w(y), \\ \dot{y} = v\sin\theta, \\ \dot{\theta} = u. \end{cases} \tag{1}$$

where x, y are horizontal and vertical coordinates of the particle, respectively, v is the velocity of the particle relative to the medium, θ is the heading angle, subjected to state constraints $\theta(t) \in [\theta_1, \theta_2]$, θ_1, θ_2 are constants, $w(y)$ is the drift in the horizontal direction, depending on y, u is a control variable, unbounded piecewise continuous function.

Boundary conditions have a form:

$$x(0) = x_0, y(0) = y_0, \theta(0) \text{ is free}, y(T) = y_T \tag{2}$$

Final time T of the process is given.

The goal function is:

$$J = -x(T) \to \min_u \tag{3}$$

The problem (1) - (4) is Mayer optimal control problem.

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