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Interval frequency response of uncertain locally resonant structures

ROBERTA SANTORO[1], MATTEO MAZZEO[1*], GIUSEPPE FAILLA[2]

1. University of Messina, Department of Engineering, Villaggio S. Agata, 98166 Messina, Italy [0000-0002-9245-9702]
2. University of Reggio Calabria, Department of Civil, Energy, Environmental and Materials Engineering (DICEAM), Via Graziella, 89124 Reggio Calabria, Italy [0000-0003-4244-231X]

* Presenting Author

Abstract: The paper aims to evaluate the interval frequency response of uncertain locally resonant structures. Following a non-probabilistic approach, the parameters of the resonators are modelled as uncertain-but-bounded variables defined by lower and upper bounds. The bounds of the frequency response of the locally resonant structure are obtained by a global optimization technique as well as first-order and second-order Taylor approximations, taking full advantage of very simple and explicit expressions ensuring a remarkable reduction of computational effort. Applications on locally resonant beams and plates focus on analysing the frequency response variability due to the uncertainties.

Keywords: locally resonant structures, uncertain-but-bounded parameters, frequency response.

1. Introduction

Structures with periodically attached resonators, generally denoted as locally resonant (LR) structures, are a promising concept for vibration attenuation with several potential applications in mechanical and structural engineering [1,2]. Commonly, LR structures as beams or plates are modelled by elastic continua (using standard beam/plate theories) coupled with discrete mass-spring subsystems modelling the resonators.

In real-life applications, the parameters of the system may be affected by uncertainties due to different sources, e.g. initial manufacturing errors, model inaccuracies and performance of the elements. This variability may lead to significant variability of the response.

The present study investigates how and to which extent the response of LR structures may be affected by uncertainty in the parameters of the resonators. Specifically, within the framework of the interval analysis [3], the parameters of the N resonators of the system, namely stiffness, mass and damping, are described as *uncertain-but-bounded* variables ranging between their lower (LB) and upper (UB) bounds. The interval uncertainty in the parameters of the resonators implies that the LR system frequency response is an interval function as well.

On resorting to a finite element approach in the frequency domain, the innovative point of our study consists in calculating the transfer matrix of the LR structure from the transfer matrix of the host uncontrollated structure as modified by summing an appropriate N-rank matrix, which represents the feedback action of the resonators on the host structure and accounts for the fluctuations of the parameters of the resonators. The proposed calculation is exact and greatly simplifies the evaluation of the interval frequency response function (FRF) of the LR structure. On this basis, we will estimate the bounds of the frequency response of the LR structure by applying a global optimization technique as well as the first-order and second-order Taylor approximations, which provide very simple and explicit expressions with a significant reduction of computational effort.

2. Results and Discussion

Consider the LR Euler cantilever beam under a unit harmonic force at the free end in Fig. 1. $N = 8$ one-degree-of-freedom resonators are attached to the beam at mutual distance a (= cell length).

The beam parameters are: $E = 70$ GPa, $I = 7.85\times10^{-9}$ m^4, $\rho = 0.88$ kgm^{-1} and $a = 0.02$ m. Fig. 2 shows the bounds of FRF amplitude for the tip deflection of the beam when mass and damping of the resonators are taken as deterministic values, $m_j = 0.01$ kg and $c_j = 0.1$ Nsm^{-1}, while uncertain-but-bounded stiffness k_j^I (for $j = 1,..., N$) is assumed to vary in the range [90000, 110000] Nm^{-1}. Using the proposed framework, LBs and UBs are calculated by a global optimization technique. Results of the Monte-Carlo simulation (MCS) are included for validation.

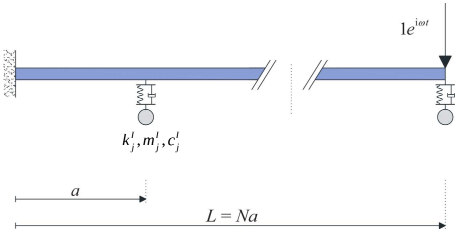

Fig. 1. LR Euler cantilever beam with one-degree-of-freedom resonators

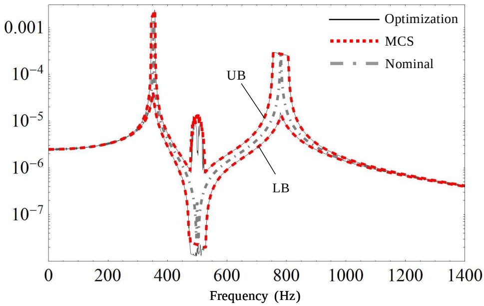

Fig. 2. LB and UB of FRF amplitude for tip deflection of LR beam in Fig. 1 (resonators with uncertain stiffness)

3. Concluding Remarks

We propose a novel computational framework to estimate the effects of uncertainties in the parameters of the resonators on the frequency response of LR structures. The framework is based on an exact expression of the transfer matrix, suitable for any finite element model of LR structures.

References

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