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A new type of undimensional optimized model for rod deduced from three dimensional elasticity

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Abstract: This paper develops a dynamic elastic linear curved rod theory consistent with three-dimensional Hamilton's principle under general loadings with a second-order error. An asymptotic reduction method is introduced to construct a curved rod theory for a general anisotropic linearized elastic material. For the sake of simplicity, the cross section is assumed to be circular. The starting point is Taylor expansions about the mean-line in curvilinear coordinates, and the goal is to eliminate the two spatial variables in the cross section in a pointwise manner in order to obtain a closed system for the displacement coefficients. We achieve this by using a Fourier series for the lateral traction condition together with the use of polar coordinates in the cross section and by considering exact tridimensional equilibrium equation. We get a closed differential system of ten vector unknowns, and after a reduction process we obtain a differential system of the vector of the mean line displacement and twist angle. Six boundary conditions at each edge are obtained from the edge term in the tridimensional virtual work principle, and a unidimensional virtual work principle is also deduced from the weak forms of the rod equations.

Keywords: curved rod theory, reduction method, anisotropic linearized elasticity, rod variational formulation, Fourier series

1. Introduction

Rods are very important engineering structures. For straight or curved rods, the dimension of the cross section is much smaller than the third one, its length. Due to this relative smallness, one may model the behaviours of these thin structures by one-dimensional rod theory through certain dimensional reduction processes. The most popular approach is to introduce some kinematic assumptions which leads to classical rod theories, such as Euler-Bernoulli rod theory, Timoshenko rod theory and well known Reddy's third-order rod theory. *In this work we propose a new optimised approach.* For plate and shell, a novel method of dimension reduction, deduced from tridimensional equilibrium equation and boundary condition on the upper and lower faces of the plate or the shell, is introduced in [1], [2], [3], [4]. This approach was used in [5] to obtain a plane-stress rod model for a linearized isotropic elastic material with pointwise error estimates. In this way [6] obtains a unidimensional model for a rod with rectangular cross section. Obtention of a rod theory is more complicated than plate or shell ones. For rod with circular cross section, we present a one dimensional model as already done in a previous work for straight rod [7] and curved rod [8]. In this paper we extend the work [8] to dynamical curved rod.

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2. Results

By using the 3-D Hamilton's principle and the differential system, we can derive the dynamic linear curved elastic rod theory which is consistent with the 3-D weak formulation (as it is made for plate in [2] and shell in [3]). We can easily obtain this model by replacement in [8], the vector coefficients of the asymptotic expansion of the body force

$$\mathbf{f}^{(k,\,n\text{-}k)} \text{ by } \mathbf{f}^{(k,\,n\text{-}k)} - \rho\ddot{\mathbf{u}}^{(k,n-k)} \quad \text{for } n = 0, 5 \text{ and } k = 0, n. \tag{1}$$

in which ρ is the mass density of the rod material and $\ddot{\mathbf{u}}^{(k,n-k)}$ is the second time derivative of the coefficient of asymptotic expansion of the displacement field (see equation (24) in [8]).
By considering remark (1), we obtain the differential system of equations (59)-(62) in [8] (with the use of recursive relations (39) and (54)). The Neumann type boundary conditions are given by (73)-(76) in [8] and the Dirichlet one by (77) in [8]. So we obtain the weak variational formulation (78) in [8] which provides a framework for implementing finite-element schemes.

3. Conclusion and discussion

The main purpose of this work is to provide a new asymptotic reduction method for constructing a consistent curved rod dynamic theory for linearized anisotropic elastic material. The starting point of our derivation is a Taylor–Young expansion of the displacement field. Then we consider the corresponding expansion of the deformation gradient and the stress tensor and make some development which are needed for the success of our procedure. More precisely we summarize the main idea. To write lateral boundary condition we use polar coordinates in cross section together with Fourier series expansion. This leads to seven equations with fifteen unknown coefficients of the displacement field. Equilibrium equations give supplementary relations between stress coefficients and we show that it is possible to obtain a closed system of ten equations with ten unknows. The linear dependence with respect to three second order displacement coefficients is used to eliminate them. Elaborated calculations furnish bending and torsion terms and also lead to asymptotically-consistent closed three vector equations with three unknwons. Then we formulate the unidimensional rod virtual work principle. Now, we have a working paper for the case of linear elastic rod with double symmetric cross section and it would be applicable to other rod problems : geometric and/or material nonlinearity/incompressible material, multilayered rod.

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References

[1] DAI H.H, SONG Z. On a consistent finite-strain plate theory based on three-dimensional energy principle. *Proc R Soc London, Ser A* 2014 470 (2171) doi.10.1098/rspa.2014.0494.
[2] SONG Z, DAI, H.H. On a consistent dynamic finite-strain plate theory and its linearization. *J Elast* 2016, 125(2):149–183.
[3] YU X, Fu Y, DAI H.H., A refined dynamic finite-strain shell theory for incompressible hyperelastic materials: equations and two-dimensional shell virtual work principle. *Proc. R. Soc. A* 476: 20200031.
[4] WANG F.F, STEIGMANN D.J, DAI H.H. On a uniformly-valid asymptotic plate theory. *Int J Non Linear Mech* 2019, 112:117–125.
[5] CHEN X, SONG Z, DAI H.H. Pointwise error estimate for a consistent beam theory. *Anal Appl* 2018, 16(01):103–132.
[6] PRUCHNICKI E. Contribution to beam theory based on 3-D energy principle. *Math Mech Solids* 2018, 23 (5):775–786.
[7] CHEN X, DAI H.H, PRUCHNICKI E. On a consistent rod theory for a linearized anisotropic elastic material: I. Asymptotic reduction method. *Math Mech Solids* 2021, 26(2) : 217–229.
[8] PRUCHNICKI E, CHEN X, DAI H.H. New refined model for curved linear anisotropic rods with circular cross section. *Applications in Engineering Science* 2021, 6 doi.org/10.1016/j.apples.2021.100046.