2021 DSTA dys-ta.com
16th INTERNATIONAL CONFERENCE
Dynamical Systems – Theory and Applications
December 6-9, 2021, On-line

# A unified Bayesian formulation for the identification of force sources

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**Abstract:**

Structures undergo some mechanical impacts during their life phase, which may create high vibration levels that induce damage and failure of the system itself. However, the mechanical characteristics of the shock are often not well known and inverse methods are generally used to estimate these complex sources. Among all the existing methods, Kalman filtering provides a lightweight and elegant solution to solve force reconstruction problems in time domain. In the literature, several Kalman-like filters have been proposed. We demonstrate in this contribution that all these formulations can be unified by expressing them from a Bayesian perspective.

**Keywords:** Inverse problem, Force reconstruction, Kalman Filter, Structural dynamics.

## 1. Introduction

Kalman-like filtering is an attractive way of solving joint input-state estimation problems in the time domain. Over the last 20 years, various Kalman-based techniques have been proposed. From an algorithmic standpoint, each method exhibits some clear differences. From a theoretical perspective, however, it can be shown that all these methods can be derived from a unique Bayesian formulation of the problem.

## 2. Results and Discussion

The space-state representation of problem that is intended to solve, is given by:

$$\begin{cases} \boldsymbol{x_{k+1}} = \boldsymbol{A}\,\boldsymbol{x_k} + \boldsymbol{B}\,\boldsymbol{u_k} + \boldsymbol{w_k} \\ \boldsymbol{y_k} = \boldsymbol{C}\,\boldsymbol{x_k} + \boldsymbol{D}\,\boldsymbol{u_k} + \boldsymbol{v_k} \end{cases} \tag{1}$$

where $\boldsymbol{x_k}$, $\boldsymbol{u_k}$ and $\boldsymbol{y_k}$ are the state, input and output vectors at sample k, while $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$ and $\boldsymbol{D}$ are the system matrices. Here, $\boldsymbol{w_k}$ (resp. $\boldsymbol{v_k}$) denotes the Gaussian process noise (resp. measurement noise) with zero mean and covariance matrix $\boldsymbol{Q_k^x}$ (resp. $\boldsymbol{R_k}$).
From a Bayesian perspective, $\boldsymbol{x_k}$ and $\boldsymbol{y_k}$ are considered as random variables defined such that $\boldsymbol{x_k} \sim p(\boldsymbol{x_k}|\boldsymbol{x_{k-1}}, \boldsymbol{u_{k-1}})$ and $\boldsymbol{y_k} \sim p(\boldsymbol{y}_k|\boldsymbol{x}_k, \boldsymbol{u_k})$. In the context of joint input-state estimation problems, additional assumptions must be made on the input vector $\boldsymbol{u_k}$.
A first idea consists in including in the state-space representation (1) the fictive state equation $\boldsymbol{u_{k+1}} = \boldsymbol{u_k} + \boldsymbol{w_k^u}$ ($\boldsymbol{w_k^u}$ is a Gaussian noise with zero mean and covariance matrix $\boldsymbol{Q_k^u}$) to define an augmented state vector and performing the input-state estimation from a standard Kalman filter. This approach, known as AKF (for Augmented Kalman Filter), provides accurate estimates of the mean of the state and input vectors, but with large uncertainties on these estimated quantities [1]. Another option consists in making an assumption on the input vector $\boldsymbol{u_k}$ predicted from all the data measured until the previous time step $k-1$, namely $\boldsymbol{y_{1:k-1}}$. A careful analysis of the existing literature shows that the state-of-art

Kalman-like filters are all based on the assumption that $\boldsymbol{u_k}$ given $\boldsymbol{y_{1:k-1}}$ follows a Gaussian distribution with mean $\boldsymbol{m_k}$ and covariance matrix $\boldsymbol{P_k}$, that is:

$$p(\boldsymbol{u_k}|\boldsymbol{y_{1:k-1}}) = N(\boldsymbol{u_k}|\boldsymbol{m_k},\ \boldsymbol{P_k}) \quad (2)$$

For instance, Sedehi considers $\boldsymbol{m_k} = \boldsymbol{0}$ and $\boldsymbol{P_k} = \boldsymbol{P^u_{k-1}}$, corresponding to the covariance matrix of the excitation estimated at $k-1$ [2], whereas Gillijns and de Moor (GDM) considers an excitation uniformly distributed over the structure (equivalent to a Gaussian distribution with zero mean and infinite covariance matrix) [3]. Finally, the Dual Kalman Filter (DKF) is composed of two classic Kalman filters running sequentially: prediction/estimation of the input vector followed by prediction/estimation of the state vector [4]. In this case, the authors consider $\boldsymbol{m_k} = \boldsymbol{u_{k-1}}$ and $\boldsymbol{P_k} = \boldsymbol{Q^u_{k-1}} + \boldsymbol{P^u_{k-1}}$.

All the previously estimation methods allow properly identifying both the location and the time history of the excitation, as shown in figure 1, presenting the estimation of a hammer impact exerted on a simply supported beam from a set of acceleration measurements.

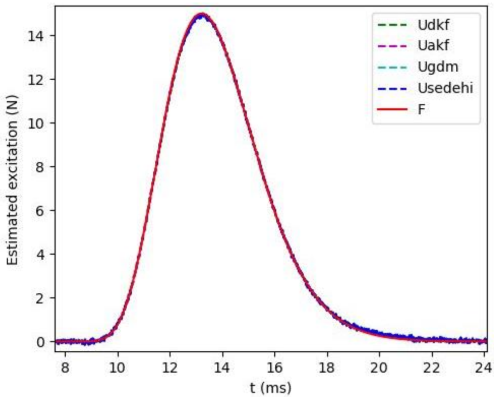


**Fig. 1.** Force identification for AKF, DKF, GDM and Sedehi methods.

## 3. Concluding Remarks

In this contribution, the Bayesian paradigm has been adopted to derive a unified vision of the state-of-the-art joint input-state estimation methods, classically used in structural mechanics. This unified Bayesian formulation points out the main differences between these identification strategies, which allows to explore some new assumptions and consequently propose original methods.

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