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Continuation analysis of overhung rotor bouncing cycles with smooth and contact nonlinearities

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Abstract:

The "brute force" time simulation approach for understanding the wide range of nonlinear phenomena linked to rotating machinery is inefficient because of the requirement for the transients to die out for every case, the need to examine multiple initial conditions, and the inability to trace unstable responses. With rotor-to-stator contact in the definition of the model, the accurate location of the contact places yet more computational load on the time simulations. Instead, a more standardised approach is used here by employing numerical continuation. Smooth nonlinearities such as cubic stiffness can give similar patterns of bouncing cycles to those seen with discontinuous stiffness rotor-stator contact models. Therefore, a 2-dof overhung rotor with cubic stiffness nonlinearity replacing the snubber contact is investigated in the rotating frame, which is used as a base to find the response of a smooth-contact definition using a hyperbolic tangent function. This work not only provides more insight into such responses, but also demonstrates numerical continuation as a potential tool to explore the nonlinear rotating system's response in a more structured way.

Keywords: Rotor-stator contact, nonlinearity, internal resonance, numerical continuation

1. Introduction

Understanding the sustained intermittent rotor-to-stator contact response in rotating machinery is important as it might be dangerous in operating conditions. Zilli et al. [1] interpreted the internal resonance on a 2-dof overhung rotor with phasors. Shaw et al. [2] defined this relation as an internal resonance condition and generalised the idea to multi-dof models. Some authors modelled the contact with an impact definition [3]. An ongoing study has revealed the similarity of bouncing cycles between cubic and discontinuous stiffness models [4]. In the present study of the nonlinear overhung rotor shown in the left of Fig. 1, the rotor-stator contact interaction is initially replaced by an isotropic cubic stiffness. Once the solution branches are known, a homotopy to a hyperbolic tangent function approximating a contact nonlinearity is applied, to explore the characteristics of the system with contact and gain new insights.

2. Models and methods

Equation (1) shows the nondimensionalised nonlinear equations of motion of the 2-dof overhung rotor in rotating coordinate frame (Fig. 1).

$$U'' + \left(-\hat{\Omega}J\left(\hat{I}_p - 2\right) + 2\zeta\right)U' + \left(\hat{\Omega}^2\left(\hat{I}_p - 1\right) + 1 + 2\zeta\hat{\Omega}J\right)U + \kappa\, f_{snub} + (\kappa - 1)f_{cubic} = \hat{m}\hat{e}\hat{\Omega}^2 \begin{Bmatrix}1\\0\end{Bmatrix}, \qquad J = \begin{bmatrix}0 & -1\\1 & 0\end{bmatrix} \tag{1}$$

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where, U is the position vector in the rotating frame, ζ is damping ratio, $f_{snub} = 0.5\tanh(K(\hat{r}-1)+1)\,\beta(\hat{r}-1)$ and $f_{cubic} = \gamma\hat{r}^2 U$ are the smooth contact and cubic stiffness restoring forces, respectively; K is the steepness of the tanh function, β is the measure of snubber-ring stiffness, and γ is the measure of cubic stiffness.

3. Results and Discussion

A converged 2:1 and 3:1 internal resonance response from the cubic stiffness simulations is used as a starting point for the numerical continuation scheme, which yields an entire solution branch. An orbit on this branch is used to start continuation of the homotopy parameter switching to the contact definition. The results are shown in the right of Fig. 1. The smooth-contact case shows period doubling, which would in a "brute force" analysis, cause difficulty in obtaining the whole branch. Period doubling bifurcations can be seen on the smooth-contact model, towards lower amplitudes. Note that the lowest amplitude level for the 3:1 cubic branch (bold green) is below the contact level; after the nonlinearity switch (thin green), the lowest point moves upward above the contact level. Although close to contact amplitude the response is mostly unstable, in the small region between nondimensional rotor speeds of 4.4 – 5.0, the unstable periodic orbits of 3:1 might push the transitional responses to the 2:1 stable periodic orbits.

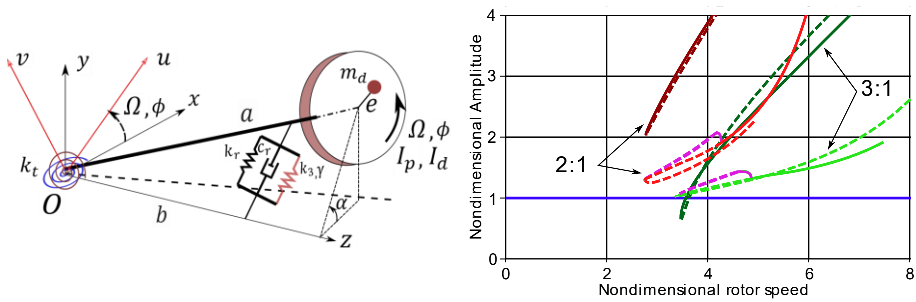

Fig. 1. Left: Schematic representation of the cubic stiffness model. Right: Bifurcation diagram with the cubic (bold lines) and with tanh nonlinearity (thin lines). Purple branches show period doubling response. Dashed lines show unstable responses. Blue line shows the contact amplitude.

This method enables one to start from an easier nonlinearity and explore the response in a more complex one without the expensive of brute force simulations, which does not guarantee even to find all stable solution responses, while completely diverging from the unstable periodic orbits.

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