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Advanced Computational Modelling Complex Dynamical Systems: The Earth Angular Momentum Balance Nonstationary Model

ALEXANDER V GLUSHKOV[1], OLGA Y KHETSELIUS[1], VALERY F. MANSARLIYSKY[1], ARTEM V. VITAVETSKY[1*]

1. Odessa State Environmental University, Mathematics Depr., L'vovskaya str. 15, 65009, Odessa
* Presenting Author

Abstract: The paper presents the elements of new mathematical formalism to computational modelling complex dynamical systems, in particular, such as planetary balance of angular momentum of the Earth. The approach provides a correct treatment of global mechanisms in the balance of the angular momentum of the Earth, macroturbulent atmospheric low-frequency processes, including processes of heat-mass transfer at spatial and temporal macro scales, teleconnection effects etc. The methods of a plane complex geophysical field and spectral expansion algorithms are applied to describe the circulation processes. The detailed description of the computational algorithm with accounting for the macro turbulent, circulation low-frequency processes is presented. The results of the PC simulation experiments on calculating a balance of planetary angular momentum (including an atmospheric part) are presented for a whole Pacific ocean region.

Keywords: nonlinear dynamics, planetary balance of angular momentum, numerical modelling

1. Introduction. Nonlinear Dynamics of Atmospheric Ventilation

The mathematical modelling nonlinear chaotic (macroturbulent) large scaled low-frequency processes in different complex dynamical system attracts a great interest because of a principal importance of such studies. From the other side, as a rule, such modeling encounters colossal computational difficulties, and sometimes the only way out is to find optimal mathematical approaches and to create effective computing algorithms for analyzing and modeling of complex dynamical systems. One of the most complex problem in theory of planetary dynamical systems is modelling planetary balance of angular momentum of the Earth as well as large scaled macroturbulent atmospheric processes. In our work starting from the key positions of a dynamical systems theory and nonlinear computational hydrodynamics the fundamentals of a new mathematical formalism to computational modelling complex dynamical systems, in particular, balance of angular momentum of the Earth are presented.

2. Model, Results and Discussion

An advanced non-stationary angular momentum balance equation can be written as follows [1,2]:

$$\frac{\partial}{\partial t}\int \rho M dV = \int_{\varphi_1}^{\varphi_2}\int_0^H\int_0^{2\pi} \rho v M d\varphi dz d\lambda + \int_0^H\int_{\varphi_1}^{\varphi_2}\int_0^{2\pi}\left(p_E^i - p_W^i\right) a\,\mathbf{cos}\,\varphi dz d\varphi d\lambda + \tag{1}$$

$$+\int_{\varphi_1}^{\varphi_2}\int_0^{2\pi}\int_0^H \tau_0 a\,\mathbf{cos}\,\varphi d\varphi d\lambda 2\pi,$$

where M = $\Omega a^2 \mathbf{cos}\varphi$ + ua**cos**φ - angular momentum; Ω - the angular velocity of rotation of the Earth; a - radius of the Earth; φ - Latitude (φ_1 - φ_2 - separated latitudinal belt between the Arctic and polar fronts); λ - longitude; u, v - - zonal and meridional components of the wind speed; ρ - air density; V - the entire volume of the atmosphere in this latitude belt from sea level to the average height of the elevated troposphere waveguide - H; $p_E^i - p_W^i$ - the pressure difference between the eastern and western slopes of the i-th mountains; z - height above sea level; τ_0 - the shear stress on the surface. From the point of view of physics, the cycle of balance of angular momentum in the contact zones with the hydrosphere and lithosphere becomes a singularity. This singularity can be detected through the occurrence of zones of fronts and soliton-type front. Then the kernel of equation (1) can be defined in the density functional ensemble of complex velocity potential [1,3]

$$w = \overline{v_\infty} z + \frac{1}{2\pi}\sum_{k=1}^{n} q_k \ln(z - a_k) + \frac{1}{2\pi}\sum_{k=1}^{p} \frac{M_k e^{\alpha_k i}}{z - c_k} - \frac{i}{2\pi}\sum_{k=1}^{m} \Gamma_k \ln(z - b_k) \qquad (2)$$

and the complex velocity, respectively, will be

$$v = \frac{dw}{dz} = \overline{v_\infty} + \frac{1}{2\pi}\sum_{k=1}^{n} \frac{q_k}{z - a_k} - \frac{1}{2\pi}\sum_{k=1}^{p} \frac{M_k e^{\alpha_k i}}{(z - c_k)^2} - \frac{i}{2\pi}\sum_{k=1}^{m} \Gamma_k /(z - b_k), \qquad (3)$$

where w – complex potential; v_∞ – complex velocity general circulation background (mainly zonal circulation);b_k – coordinates of vortex sources in the area of singularity; c_k – coordinates of the dipoles in the area of singularity; a_k – coordinates of the vortex points in areas of singularity; M_k – values of momenta of these dipoles;α_k – orientation of the axes of the dipoles; Γ_k, q_k – values of circulation in the vortex sources and vortex points, respectively. Besides, it is necessary to have a effective method for modelling macroturbulent atmospheric processes [3-5]. It is used an advanced version of the standard tensor equations of turbulent tensions. As usually, it is convenient to partition velocity $\boldsymbol{u}(v_x,v_y,w)=(U,V,W)$, pressure p, temperature θ into equilibrium and departures from equilibrium values (for example: $p=p_o+p'$etc). The total system includes equations for the Reynolds tensions, moments of connection of the velocity pulsations with entropy ones and the corresponding closure equations. The technique of using Reynolds tension tensors of the second rank is well known (for example, in the form of an analytical representation). The results of calculating the balance of angular momentum, atmospheric circulation in link with continuity of atmospheric circulation forms are presented for a whole region of Pacific ocean. The numerical data for the a current function and a complex velocity potential are listed.

References

[1] GLUSHKOV A., KHETSELIUS O., AMBROSOV S., BUNYAKOVA YU., MANSARLIYSKY V., The use of microsystems technology "Geomath" to modeling the balance of the angular momentum of the Earth, atmospheric processes and parameters of radio waveguides. *Sensor Electr. and Microsyst. Techn.* 2013, **10**(1):22-28.

[2] GLUSHKOV A, KHETSELIUS O, SVINARENKO A, BUYADZHI V: *Methods of computational mathematics and mathematical physics.* P.1. TES: Odessa, 2015

[3] KHETSELIUS O, GLUSHKOV A, STEPANENKO S, SVINARENKO A, BUYADZHI V: Nonlinear dynamics of the industrial city's atmospheric ventilation: New differential equations model and chaos. In: AWREJCEWICZ J (Ed.) *Perspectives in Dynamical Systems III: Control and Stability*. Series: *Springer Proceedings in Mathematics & Statistics*. 2021, **364**:Ch.16

[4] RUSOV V, GLUSHKOV A, VASCHENKO V ETAL**:** Galactic cosmic rays – clouds effect and bifurcation model of Earth global climate. Part 1. Theory. *J. of Atmos. and Solar-Terrestrial Phys*. 2010, **72**:498-508.

[5] KHETSELIUS O: Forecasting evolutionary dynamics of chaotic systems using advanced non-linear prediction method. In: AWREJCEWICZ J, KAZMIERCZAK M, OLEJNIK P AND MROZOWSKI J (EDS.) *Dynamical Systems Applications.* Politechniki Łódzkiej: Łódz, 2013, **T2**:145-152.