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Simulation of non-linear coupled dynamic systems of first and second order applying a semi-analytic method

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Abstract: Mechanical models for non-linear dynamic systems are defined by differential equations of second order where also first order differential equations are present frequently. Differential equations of first order are present if some degrees of freedom have no corresponding mass, if the stiffness parameters vanish in a part of the equations or if a controller is implemented in the system. For linear systems of first and second order various numerical procedures for solving the differential equations are available. A semi-analytical method is presented which is exact for the linear dynamic and decoupled systems of first and second order. A modal transformation of the partitioned system equations is necessary for each part. After a discretization in the time-domain the relevant equations for a suitable and effective time-integration algorithm are defined taking the non-linearity into account. The resulting procedure is derived and it turns out that the formulation is analogous to a BEM-formulation in time as Green's functions are used. The method is extended to the coupled non-linear differential equations of first and second order and is applied to a system with two degrees of freedom.

Keywords: first and second order systems, time-integration, non-linear dynamics

1. Introduction to dynamic systems of first and second order

The differential equations of motion for non-linear dynamic system are given in the form

$$\boldsymbol{M}\ddot{\boldsymbol{X}} + \boldsymbol{D}\dot{\boldsymbol{X}} + \boldsymbol{K}\boldsymbol{X} + \boldsymbol{F}_N = \boldsymbol{F}(t). \tag{1}$$

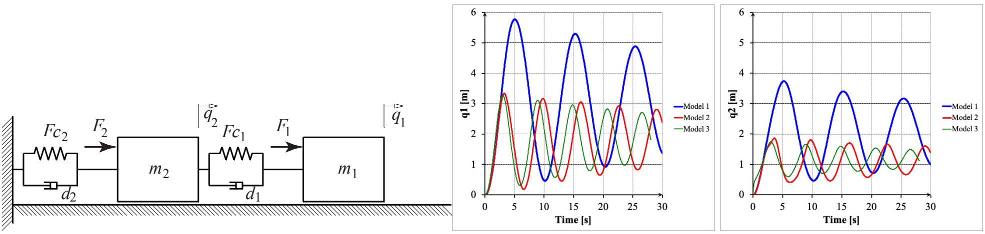
The non-linear equations of motion with the mass matrix $\boldsymbol{M}$, the damping matrix $\boldsymbol{D}$, the stiffness matrix $\boldsymbol{K}$, the vector of non-linear reaction force $\boldsymbol{F}_N$, the vector of excitation force $\boldsymbol{F}(t)$ and the vector of degrees of freedom $\boldsymbol{X}(t)$ with n components. For mechatronic system the equations for dynamic systems frequently include first and second order differential equations. Examples for such systems are the temperature in a body with heat exchange, the balance of mass with inflow and outflow and control of dynamic systems. For such systems the solution is computed frequently after a transformation into a system of differential equations of first order. For linear systems of first and second order a variety of solutions is presented in [1] and [2] and various numerical procedures for solving the differential equations in [3] and [4]. Due to the high effort of this method specially for Finite Element models a semi-analytic method is presented. The equations of motion can be partitioned and with the assumptions that $\boldsymbol{M}_{12} = \boldsymbol{0}_{12}, \boldsymbol{M}_{21} = \boldsymbol{0}_{21}, \boldsymbol{M}_{22} = \boldsymbol{0}_{22}$ after additional manipulations it follows

$$\begin{bmatrix} \boldsymbol{M}_{11} & \boldsymbol{0}_{12} \\ \boldsymbol{0}_{21} & \boldsymbol{0}_{22} \end{bmatrix} \begin{Bmatrix} \ddot{\boldsymbol{X}}_1 \\ \ddot{\boldsymbol{X}}_2 \end{Bmatrix} + \begin{bmatrix} \boldsymbol{D}_{11} & \boldsymbol{0}_{12} \\ \boldsymbol{0}_{21} & \boldsymbol{D}_{22} \end{bmatrix} \begin{Bmatrix} \dot{\boldsymbol{X}}_1 \\ \dot{\boldsymbol{X}}_2 \end{Bmatrix} + \begin{bmatrix} \boldsymbol{K}_{11} & \boldsymbol{0}_{12} \\ \boldsymbol{0}_{21} & \boldsymbol{K}_{22} \end{bmatrix} \begin{Bmatrix} \boldsymbol{X}_1 \\ \boldsymbol{X}_2 \end{Bmatrix} = \begin{Bmatrix} \boldsymbol{F}_1 \\ \boldsymbol{F}_2 \end{Bmatrix} - \begin{Bmatrix} \boldsymbol{F}_{\mathrm{N}1} \\ \boldsymbol{F}_{\mathrm{N}2} \end{Bmatrix}$$
$$- \begin{bmatrix} \boldsymbol{0}_{11} & \boldsymbol{D}_{12} \\ \boldsymbol{D}_{21} & \boldsymbol{0}_{22} \end{bmatrix} \begin{Bmatrix} \dot{\boldsymbol{X}}_1 \\ \dot{\boldsymbol{X}}_2 \end{Bmatrix} - \begin{bmatrix} \boldsymbol{0}_{11} & \boldsymbol{K}_{12} \\ \boldsymbol{K}_{21} & \boldsymbol{0}_{22} \end{bmatrix} \begin{Bmatrix} \boldsymbol{X}_1 \\ \boldsymbol{X}_2 \end{Bmatrix} \tag{2}$$

For the first part of a differential equation of second order a modal analysis and transformation is performed to get diagonal matrices of the first partition. For the second part of a differential equation of first order an eigenvalue computation is done and decoupled equations follow after the corresponding transformation. The coupling forces of this modified equations are considered in the convolution integral. With a linear interpolation of the generalized coordinates an implicit procedure results. The non-linear force within the time step are assumed to be a linear function of time. After a reformulation to an incremental and iterative Newton-Raphson-procedure results. The formulation of the derived procedure is analogous to the BEM-formulation in time, described in [5] for a system of second order, where the formulation of the system of first order and the coupling of the sub-systems is considered.

2. Results and Discussion

The computation is demonstrated for a dynamic system with two degrees of freedom. In this special case the eigenvalue analysis is very simple and the results can be studied for different parameters represented in three different models. The non-linearity of the springs for the mechanical model of Fig. 1a are given by $F_{c1} = c_1(q_1 - q_2) + c_3(q_1 - q_2)^3$ and $F_{c2} = c_2 q_2 + c_3 {q_2}^3$. The parameters for the *linear Model 1* are $m_1 = m_2 = 1$ kg, $d_1 = d_2 = 0.1$ Ns/m, $c_1 = c_2 = 1$ N/m, $c_3 = c_4 = 0$ N/m^3 and $F_1 = F_2 = 1$ N. The *non-linear Model 2* differs from Model 1 by $c_3 = c_4 = 0.5$ N/$m^3$and the *non-linear Model 3* differs from Model 2 by the mass $m_2 = 0$ kg. In Fig. 1b and 1c the computed solution is shown for the three models. A comparison to conventional time integration algorithms shows a better performance of the present method. Analogous to the analysis of the semi-analytic time-integration method for variable mass non-symmetric and non-linear systems in [4] the present method is analyzed with respect to the numerical behavior. The numerical behavior of the algorithm is analyzed for a defined non-linearity and additionally the stability and accuracy of the algorithm are studied. The resulting algorithm shows a high efficiency when compared with well-known numerical methods.



a b c

Fig. 1. Mechanical Model and solution of the mechanical system with homogeneous initial conditions

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