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Non-smooth dynamics of a bouncing golf ball

STANISŁAW W. BIBER[1*], ALAN R. CHAMPNEYS[2], ROBERT SZALAI[3]

1. Department of Engineering Mathematics, University of Bristol, Bristol, UK [0000-0001-5157-2904]
2. Department of Engineering Mathematics, University of Bristol, Bristol, UK [0000-0001-7772-3686]
3. Department of Engineering Mathematics, University of Bristol, Bristol, UK [0000-0002-3602-2397]
* Presenting Author

Abstract: We present a problem regarding the mathematical modelling of a bouncing golf ball. Experimental data shows a significantly different behaviour of the ball during the bounce depending on whether it slips throughout the whole time is in contact with a bouncing surface or if it enters rolling (grips the surface) at any stage of the bounce. We present a simple piecewise-smooth linear model of the compliant ground based on Kelvin-Voigt mechanism and analyse its dynamics. Assuming Coulomb friction between the ball and the ground a discontinuity arises in the equations of motions and thus one evaluates the dynamics of the mechanism treating it as a Filippov system. We recognise numerous behaviours that can arise from such model which agree with the commonly known outcomes in the game of golf.

Keywords: piecewise-smooth dynamics, Coulomb friction, Filippov systems

1. Introduction

The bounce of a ball in sports such as tennis, cricket or football has been studied extensively with many experimental data available to support the analysis. The common denominator for these models is an impact of a compliant ball off a rigid ground. A bounce of a golf ball is a very different problem, where the analysis focuses on the impact of a rigid body off a compliant surface. Previous studies have either focused heavily on obtaining and interpreting specific data [1] or presented models for the bounce but with little experimental validation [3] leaving a large gap between scientific models and data. In this project we aim at developing the fundamental understanding behind the science of the golf ball bounce by presenting an appropriate model for the behaviour of a compliant golf turf which we will aim to support by experimental validation.

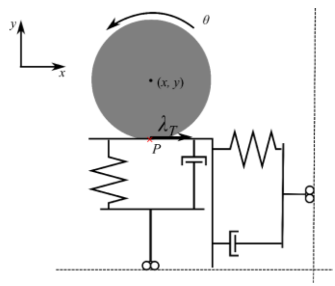

Fig. 1. Illustration of the compliant ground model

We consider a setting in which a rigid sphere of unit radius impacts a compliant surface based on Kelvin-Voigt mechanism – see Fig. 1. for the illustration of the mechanism. Denoting the centre of mass of the ball with (x, y) and its spin with $\boldsymbol{\omega}$ the equations or motion are

$$\ddot{x} + \frac{2d_x}{\varepsilon_x}\dot{x} + \frac{1}{\varepsilon_x}x = \lambda_T, \quad \ddot{y} + \frac{2d_y}{\varepsilon_y}\dot{y} + \frac{1}{\varepsilon_y}y = -g, \quad \dot{\omega} = \frac{5}{2}\lambda_T, \tag{1}$$

where λ_T is the tangential force governed by the Coulomb friction law. We introduce the vector of state variables $p = [x, \dot{x}, y, \dot{y}, \omega]^\top$, and thus the equations of motion are rewritten as

$$\dot{p} = \begin{cases} F_1 p + g & \text{if} \quad H(p) > 0, \\ F_R p + g & \text{if} \quad H(p) = 0, \\ F_2 p + g & \text{if} \quad H(p) < 0, \end{cases} \tag{2}$$

where $F_1, F_R, F_2 \in \mathbb{R}^{5\times5}$, $g = [0, 0, 0, -g, 0]^\top$ and $H(p) = \dot{x} + \omega$. This gives us three discontinuous, piecewise-smooth differential equations, where the switching condition is the velocity of the point of contact of the ball. Situations where $H(p) > 0$ and $H(p) < 0$ correspond to slipping with positive and negative tangential velocities at the point of contact respectively, whereas the ball is rolling when $H(p) = 0$.

Such system with a discontinuous first derivative is known as a *Filippov system*. In the analysis of this system we focus on the possible dynamics of the ball after it enters rolling, with a particular focus on whether the ball can enter slip again.

2. Results and Discussion

For a golf turf we select $\varepsilon_y \ll \varepsilon_x$, which means that the ground is a significantly more compliant in the normal direction than it is in the horizontal one. Alternative choices will lead to unphysical results, a particular example of which is $\varepsilon_x \approx \varepsilon_y$, in which both directions behave similarly and thus the lift off trajectories follow the inbound ones. In the physical setting the ball will enter the bounce slipping and may either change the direction from positive to negative slip without entering rolling, may slip throughout the impact or will enter rolling at some point. If the ball enters rolling it may then enter slipping again, but *only with a negative tangential velocity* of the contact point. It is also possible for the ball to remain rolling for the remaining part of impact, in which case an appropriate modelling framework, such as Utkin equivalent control law, must be applied to understand how the ball can remain on the boundary between the rolling surface and the negative slip region [2].

3. Concluding Remarks

The model of a compliant ground we propose presents a number of relevant behaviours that can be observed in the game of golf. Experimental campaign carried out in a controlled environment showed these to be applicable, however the data set is soon to be expanded by data from regular golf turf.

A problem that remains is the one of fitting parameters to the data. This proves to be difficult due to the discontinuities in the model which depend on the parameters themselves. We are in process of developing a structured approach which will see the use methods from finite element analysis and linear complementarity problems.

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