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Hidden dynamics of maps (and when "period 2 implies chaos")

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The term `hidden dynamics' describes behaviour associated with discontinuities in nonsmooth dynamical systems, behaviour this is only evident by careful study of the discontinuity itself. First introduced in continuous time flows in 2013, hidden dynamics was only found in discrete time systems last year in [1]. Hidden periodic orbits have been shown to `fill the gaps' in bifurcations and period adding cascades, so that rather than attractors in nonsmooth maps being able to appear and disappear or change period seemingly arbitrarily, as seemed to be the case until now, they in fact follow familiar bifurcations such as folds and flips well known from continuous systems.

In a discrete time system, a hidden orbit is an infinitely unstable trajectory that involves at least one iterate lying on a point of discontinuity. Hidden orbits can form periodic or chaotic structures much like regular orbits. Like standard unstable orbits, they help organise bifurcation structures and basins of attraction. By accounting for hidden orbits, maps with discontinuities can be shown to obey the rule that `period 3 implies chaos' (or Sharkovskii's theorem more generally) that until now appeared to require a map to be continuous.

In fact, as we will show here, nonsmooth systems obey a surprising form of this result, namely that `period 2 implies chaos'. We will show the conditions under which this implies, and by considering a nonsmooth map to be the singular limit of a continuous map, show how one interprets hidden orbits as unstable structures, and show that if we smooth out a discontinuity, a period 2 orbit perturbs to form orbits of period 3 (and hence all periods in accordance with Sharkovskii's theorem).

Finally, for the first time, we will show hidden orbits in applied models, taking a model of the homeostatic cycles between sleep-wake transitions, a model of growth and mitosis in yeast cells, and in a toy model of such processes as a 2-dimensional nonsmooth oscillator.

References

[1] M. Jeffrey and S. Webber. The hidden unstable orbits of maps with gaps. Proc. R. Soc. A, 476(2234):20190473, 2020