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Dynamics of railway wheelsets with a nonsmooth contact force model

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Abstract: The dynamics of a railway wheelset is investigated, focusing on the effect of the contact force models. For large values of creep velocities, the Coulomb model can be used as an asymptotic approximation of wheel-rail contact forces. Then, we get a nonsmooth dynamical system with codimension–2 discontinuities. At the discontinuity of the phase space, the so-called limit directions can be found, which correspond to the possible transitions between slipping and rolling. By this analysis, the nonsmooth model can complement the usual linear creep force model from the opposite approach, and we can explore more details about the qualitative behaviour of the wheelset.

Keywords: railway wheelset, Coulomb friction, nonsmooth dynamics

1. Introduction

Consider a wheelset of a vehicle running with a constant speed v. In the literature (see, e.g. [1] and [2]), the oscillating motion of the wheelset is often described by the lateral displacement y and the yaw angle ψ (see Fig. 1). Then, the so-called creep velocities u_x and u_y can be defined from the linearised kinematics,

$$u_x = -u_x^+ = u_x^- = b\dot{\psi} + \frac{vh}{r} \cdot y, \qquad u_y = u_y^+ = u_y^- = \dot{y} - v\psi, \tag{1}$$

where the conicity h of the wheelset is assumed to be small.

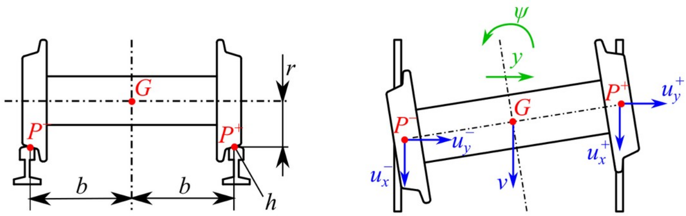

Fig. 1. Front view (left panel) and top view (right panel) of the mechanical model.

Assume that the centre point G of the wheelset is rigidly connected to the vehicle in the longitudinal direction, while the lateral and yaw motions are supported by linear springs. Then, the equations of motion of this two–degree–of–freedom model become

$$m\ddot{y} + k_y y = 2F_{\text{lat}}(u_x, u_y), \qquad J\ddot{\psi} + k_\psi \psi = 2bF_{\text{long}}(u_x, u_y), \tag{2}$$

where m, J are the mass and mass moment of inertia of the wheelset; k_y, k_ψ are resultant stiffnesses with respect to the lateral and yaw directions; and F_{lat},F_{lat} denote the components of the tangential (creep) forces at the contact points P^+ and P^-.

2. Nonsmooth contact model as a limit case of saturation

The empirical models of creep forces F_{lat} and F_{lat} shows the tendency of *sigmoid* functions containing a nearly linear region at the origin with a slope reciprocal to the vehicle speed v, and the saturating region of the creep forces for large creep velocities (see Fig. 2). By including these two effects, we can consider the *nonlinear creep model* in the form

$$F_{\text{long}}(u_x,u_y)=\frac{-Cu_x}{\sqrt{u_x^2+u_y^2+C^2v^2/c_x^2}},\qquad F_{\text{lat}}(u_x,u_y)=\frac{-Cu_y}{\sqrt{u_x^2+u_y^2+C^2v^2/c_y^2}},\tag{3}$$

where c_x, c_y are the linear creep coefficients and C is the saturation value of the creep forces.

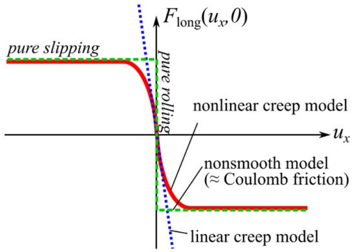

Fig. 2. The nonlinear creep characteristics (continuous line) and its approximate models.

In the limit case of *small creep velocities* in the sense $C^2v^2(u_x^2+u_y^2) \ll \min(c_x,c_y)^2$, the model (3) tends to the classical linear creep model $F_{\text{long}}(u_x,u_y)=-c_x/v\cdot u_x$ and $F_{\text{lat}}(u_x,u_y)=-c_y/v\cdot u_y$, which can be used for stability analysis as a leading–order approximation.

In the limit case of *large creep velocities* in the sense $C^2v^2(u_x^2+u_y^2) \ll \max(c_x,c_y)^2$, the nonlinear creep model (3) becomes

$$F_{\text{long}}(u_x,u_y)=-Cu_x\Big/\sqrt{u_x^2+u_y^2},\qquad F_{\text{lat}}(u_x,u_y)=-Cu_y\Big/\sqrt{u_x^2+u_y^2}.\tag{4}$$

The model (4) has the form of the Coulomb friction model, and it is *discontinuous* at $u_x=u_y=0$. *Now, we investigate the nonsmooth contact model* (4) *as an asymptotic approximation of the creep force curve to explore the qualitative behaviour of the wheelset at the saturation of the creep force curve.* For that, we apply the recently developed methods of codimension–2 discontinuities [3], and determine the possible directions of trajectories at slipping–rolling transitions. A similar method was earlier applied to the vibrations of towed wheels of road vehicles [4].

A further research direction is *to insert the possibility of creep associated with the rotation of the wheelset about its symmetry axis*. Then, the condition $u_x^+=u_x^-$ is not valid any more, and we get a nonsmooth vector field with discontinuities at the surfaces $u_x^+=u_y=0$ and $u_x^-=u_y=0$. In this case, a careful analysis is required at the intersection of these discontinuities in the of the vector field.

References

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