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On Longitudinal Oscillations in a Hoisting Cable with Time-varying Length subject to a Nonclassical Boundary Condition

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Abstract: In this work, a model of a flexible hoisting system is presented. By applying Hamilton's principle, an initial-boundary value problem for a wave equation is derived on a time-varying spatial interval with a small harmonic boundary disturbance at one end and a moving nonclassical boundary at the other end. Due to the small harmonic disturbance at one end, large resonance behavior of the system may occur. By applying an adapted version of the method of separation of variables, averaging and singular perturbation techniques, and a three time-scales perturbation method, resonances in the system are detected and accurate, analytical approximations of the solutions of the problem are constructed. It will turn out that small order ε excitations can lead to order $\sqrt{\varepsilon}$ responses. Finally, numerical simulations are presented, which are in full agreement with the obtained analytical results.

Keywords: interior layer analysis, multiple-timescales perturbation method, resonance manifold.

1. Introduction

In elevator cables, large axial oscillations can occur when a cage subject to disturbances is lifted up and down. An example of these oscillations can be founded in the mining cables, which are used to transport the cargos in a cage between the working platform and the ground. External disturbances exerted on the cage, such as airflow, can induce large vibrations and damage to the performance of the system can be caused. In order to prevent failures, it is important to understand the nature of the longitudinal vibrations of a cable with time-varying length and involving various boundary conditions. There is a lot of research on these types of problems. Gaiko and van Horssen in [1] discussed resonances and vibrations in an elevator cable system due to boundary sway. Wang et al. in [2] established a coupled dynamic modelling of the flexible guiding hoisting system and computed the response by using numerical simulations.

In this paper, a hoisting system consists of a drum, a head sheave, a driving motor, a hoisting moving conveyance and a hoisting rope with time-varying length. The upper end of the hoisting rope is fixed on the drum driven by a driving motor, which leads to the fundamental excitation. And the flexible hoisting rope lets the hoisting conveyance run up and down (Fig.1).

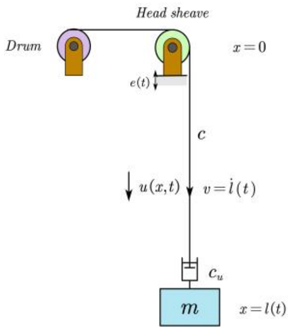

Fig. 1. The longitudinal vibrating string with time-varying length.

2. Formulation of the problem

By using Hamilton's principle, the system to describe the longitudinal vibration of a moving string as shown in Fig. 1 can be derived, and is given by:

$$\begin{cases} \rho(u_{tt}+2vu_{xt}+v^2u_{xx}+au_x+a)-EAu_{xx}+c(u_t+vu_x)=0, \quad 0\le x\le l(t),\ t>0, \\ [m(u_{tt}+2vu_{xt}+v^2u_{xx}+au_x+a)+EAu_x+c_u(u_t+vu_x)]|_{x=l(t)}=0, \quad t>0, \\ u(0,t)=e(t), \quad t>0, \\ u(x,0)=u_0(x), \quad u_t(x,0)=u_1(x), \quad 0\le x\le l(t), \end{cases} \tag{1.1}$$

where u(x, t) is the longitudinal displacement, $l(t)=f(\varepsilon t)$ is the length of the hoisting rope, v and a are the longitudinal velocity and acceleration of the hoisting rope, respectively, ρ is the linear density of the hoisting rope, m is the mass of the hoisting conveyance, EA is the longitudinal stiffness, c is the viscous damping coefficient of the hoisting rope, c_u is the viscous damping coefficient in the hoisting conveyance, and $e(t)=\varepsilon sin(kt)$ is the longitudinal fundamental excitation along the axis of the hoisting rope.

3. Methods

Firstly, since l(t) changes slowly in time, by introducing an adapted version of the method of separation of variables, the original partial differential equation can be transformed into linear ordinary differential equations with slowly varying (prescribed) frequencies. Next, the slow variation leads to a singular perturbation problem. By applying an interior layer analysis in the averaging procedure a resonance manifold is found. By using a three time-scales perturbation method, resonances in the problem are detected and accurate, analytical approximations of the solutions of the problem are constructed. It will turn out that small order ε excitations can lead to order $\sqrt{\varepsilon}$ responses.

References

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