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Nonlinear Vibration of a Functionally Graded Beam on Winkler-Pasternak Foundation Under a Moving Force

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Abstract: In this work we investigate the nonlinear thermomechanical vibrations of a functionally graded beam on Winkler-Pasternak elastic foundation. Using von Karman geometric nonlinearity, we obtain a forced nonlinear differential equation with quadratic and cubic nonlinear terms. The accuracy of the analytical results obtained by means of the Optimal Auxiliary Functions Method is proved by numerical simulations developed in order to validate the proposed procedure.

Keywords: Nonlinear vibration, functionally graded beam, Winkler-Pasternak foundation

1. Introduction

Functionally graded materials (FGM) have a wide range of applications in various fields of engineering like automotive, semiconductor industry, manufacturing industry, aerospace or defence industry. Many researchers have investigated different aspects of FGM. Nguyen and Bui [1] formulated a higher-order beam element for dynamic analysis of FGM Timoshenko beams. Soncco et al [2] utilized higher-order nodal-spectral interpolation functions to approximate the field variables minimizing the locking problem. Finite element model based on third order efficient lager-wire theory for smart FGM beam has been presented by Yasin et al [3].

In this paper we apply the Optimal Auxiliary Functions Method (OAFM) [4] to investigate dynamic behaviour of a functionally graded beam (FGB) on a Winkler-Pasternak foundation subjected to a moving force.

2. Results and Discussion

The functionally graded beam under investigation in this study is presented in figure 1.

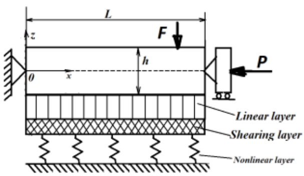

Fig. 1. Schematic of the FGB with nonlinear foundation

The functionally graded beam having the length *L*, the width *b* and the thickness *h* is resting on an elastic foundation of Winkler-Pasternak type and is subject to an axial force *P* and an axially moving force *F*. The mechanical properties of FGB can be varied as a power function

$$E(z)=(E_2-E_1)\left(\frac{z+0.5h}{h}\right)^k+E_1,\quad \rho(z)=(\rho_2-\rho_1)\left(\frac{z+0.5h}{h}\right)^k+\rho_1$$
$$\nu(z)=(\nu_2-\nu_1)\left(\frac{z+0.5h}{h}\right)^k+\nu_1 \tag{1}$$

where subscript 1 and 2 denote the top and bottom surface. Based on Euler-Bernoulli theory, we have

$$\bar{u}(x,z,t)=u(x,t)+z\frac{\partial W}{\partial x}\quad \overline{W}(x,z,t)=W(x,t) \tag{2}$$

Taking into consideration the curvature of the beam, the axial coupling and bending stiffness, the resultant force and thermal momentum, the axial force, as well as the reaction of the elastic Winkler-Pasternak foundation, the governing nonlinear thermomechanical vibration equation of FGM is

$$I\ddot{\overline{W}}+\left(D_{11}-\frac{B_{11}^2}{A_{11}L}\right)\overline{W}^{(IV)}-\overline{W}''\left[\frac{A_{11}}{2L}\int_0^L\overline{W}'^2\,dx+\frac{B_{11}}{L}\left(\overline{W}'(L,t)-\overline{W}'(0,t)\right)+N_{T\bar{x}}-P\right]=F_{\overline{W}} \tag{3}$$

where the dot and the prime denote derivative with respect to time and with respect to the variable x, respectively. After introducing dimensionless parameters and using Galerkin method, we have

$$W(x,t)=\sum_{i=1}^{N}X_n(x)T_n(t) \tag{4}$$

Further, after simple manipulations one obtains

$$\ddot{T}_n+\sum_{i=1}^{3}a_{ij}T_j+\sum_{i=1}^{3}b_{ij}T_j^3+\sum_{i\neq j=1}^{3}C_{nij}T_iT_j^2+d_nT_1T_2T_3+\sum_{i=1}^{3}e_{ni}T_i^2+\sum_{i\neq j=1}^{3}f_{nij}T_iT_j=$$
$$=F[\cosh\,p_nvt-\cos\,p_nvt-\frac{\cosh\,p_n-\cos\,p_n}{\sinh\,p_n-\sin\,p_n}(\sinh\,p_nvt-\sin\,p_nvt)] \tag{5}$$

with the corresponding initial conditions

$$T_1(0)=A,\quad T_2(0)=B,\quad T_3(0)=C,\quad \dot{T}_1(0)=\dot{T}_2(0)=\dot{T}_3(0)=0 \tag{6}$$

At this stage, the Optimal Auxiliary Functions Method (OAFM) is employed to obtain explicit analytical solutions to (5)-(6), which are in excellent agreement with the numerical integration results.

3. Concluding Remarks

The nonlinear behaviour of the considered functionally graded beam on Winkler-Pasternak foundation was comprehensively investigated by means of an efficient analytical technique, namely the Optimal Auxiliary Functions Method which proved to be very effective and accurate in solving this kind of problems.

References

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