16th INTERNATIONAL CONFERENCE
Dynamical Systems – Theory and Applications
December 6-9, 2021, On-line
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Dynamic Response of Simply-Supported Euler-Bernoulli Beam on Non-linear Elastic Foundation Under a Moving load

NICOLAE HERISANU[1*], VASILE MARINCA[2]

1. University Politehnica Timisoara, Romania
2. University Politehnica Timisoara, Romania
* Presenting Author

Abstract: The dynamic response of a Euler-Bernoulli beam with uniform cross-section resting on a nonlinear elastic foundation under a moving load is obtained by means of the Optimal Auxiliary Functions Method (OAFM). The Galerkin method is applied to discretize the nonlinear partial differential governing equation of the forced vibration and then the OAFM is employed to investigate the nonlinear behavior. This new approach provides a simple but rigorous technique to control the convergence of the solutions, which proved to be very efficient in this kind of problems.

Keywords: moving load, nonlinear vibration, Euler-Bernoulli beam, OAFM

1. Introduction

The problems related to moving loads on nonlinear foundations are very common for mechanical structures such as railway equipment, vehicle-bridge interaction, pipelines transversally supported, power transmissions. Many studies can be found on this domain. Mustafa et al. [1] studied the vibration of an axially moving beam traveling axially on a curved frictionless foundation with nonlinear elastic characteristics. The effects of a speed-dependent tension and a tension dependent speed with the inhomogeneous boundary conditions arising from Kelvin viscoelastic constitutive relations are taken into account by Tang and Ma [2]. The traveling wave modes are investigated for axially moving string and beam by Lu et al. [3]. Ri et al. [4] proposed a forced vibration model of composite beams under the action of periodic excitation force considering geometric nonlinearity. In this paper, the dynamic response of simply-supported Euler-Bernoulli beam on nonlinear elastic foundation under a moving load is investigated by means of a new approach, using the Optimal Auxiliary Functions Method (OAFM) [5].

2. Results and Discussion

A simply supported beam-type structure of length L, moment of inertia I, cross-sectional area A and modulus of elasticity E resting on nonlinear Winkler foundation is considered. Using Hamilton's principle and considering Euler-Bernoulli beam theory, the differential governing equation is derived as

$$\rho A \frac{d^2\overline{W}(\bar{x},\bar{t})}{d\bar{t}^2} + \frac{d^2}{dx^2}\left[\frac{EIW''(\bar{x},\bar{t})}{\sqrt{1-W'^2(\bar{x},\bar{t})}}\right] + PW''(\bar{x},\bar{t}) + \\ + \overline{K}_1 W(\bar{x},\bar{t}) + \overline{K}_3 W^3(\bar{x},\bar{t}) - \overline{K}_2 W''(\bar{x},\bar{t}) = \overline{F}_0 \delta(\bar{x} - \bar{v}\bar{t}) \tag{1}$$

Assuming a three-mode approach one obtains

$$\overline{W}(\bar{x},\bar{t}) = \sum_{i=1}^{3} q_i(\bar{t}) X_i(\bar{x}) \; ; \quad X_i(\bar{x}) \tag{1}$$

and applying the Galerkin method we have

$$\int_0^L [\rho A \ddot{\overline{W}} + EI(\overline{W}^{(IV)} + \frac{1}{2}\overline{W}^{(IV)}\overline{W}'^2 + 3\overline{W}'\overline{W}''\overline{W}''' + \overline{W}''^3) + \\ + (P - K_2)\overline{W}'' + K_1\overline{W} + K_3\overline{W}^3 - F_0\delta(x - vt)] X_i(\bar{x}) d\bar{x} = 0, \quad i = 1,2,3 \tag{1}$$

After introducing some dimensionless parameters, one obtains the nondimensional nonlinear differential equation

$$\ddot{q}_1 + (\pi^4 r^2 + K_1 - \pi^2 K_2) q_1 + \frac{1}{4}(5\pi^6 r^2 + 3K_3) q_1^3 + (K_3 - \pi^6 r^2) q_1 q_2^2 + \\ + \frac{1}{8}(171\pi^6 r^2 - 3K_3) q_1^2 q_3 + \frac{1}{4}(27\pi^6 r^2 + 6K_3) q_1 q_3^2 + 246\pi^6 r^2 q_2^2 q_3 = F_0\sqrt{2}\sin \pi v t \tag{1}$$

Following the same procedure, similar nonlinear equations are obtained for q_2 and q_3.

The initial conditions for the vibration of the considered simply supported Euler-Bernoulli beam based on the triple-mode assumption are

$$q_1(0) = A_1, \quad q_2(0) = A_2, \quad q_3(0) = A_3, \quad \dot{q}_1(0) = \dot{q}_2(0) = \dot{q}_3(0) = 0 \tag{1}$$

At this stage, the Optimal Auxiliary Function Method is applied to obtain analytical approximate solution of the form

$$\bar{q}_1 = q_{10} + q_{12} \; ; \quad \bar{q}_2 = q_{20} + q_{22} \; ; \quad \bar{q}_3 = q_{30} + q_{23} \tag{1}$$

where the initial approximations and the first-order approximations can be determined from the specific linear equations according to OHAM procedure, while the natural frequencies of every mode of vibration are determined by avoiding secular terms. Every term of the first-order approximation contains a set of so-called convergence-control parameters, whose values are optimally determined by rigorous procedure.

3. Concluding Remarks

The Optimal Auxiliary Functions Method was applied to comprehensively investigate the nonlinear behaviour of a simply-supported Euler-Bernoulli beam on nonlinear elastic foundation under a moving load. The capabilities of OAFM were successfully tested on a very complex problem, which does not have exact solutions and it is very difficult to be solved by traditional methods. Highly accurate explicit analytical solutions are obtained by the proposed technique, which proves its applicability in investigating systems of strongly nonlinear differential equations in the absence of small parameters.

References

[1] MUSTAFA A.M., HAWA M.A., HARDT D.E.: Vibration of an axially moving beam supported by slightly curved elastic foundation. *J. Vibr. Control* 2017, **24**(17):4000-4009.

[2] TANG Y.Q., MA Z.G.: Nonlinear vibration of axially moving beam with internal resonance, speed dependent tension and tension-dependent speed. *Nonlin. Dyn.* 2019, **98**:2475-2490.

[3] LU L., YANG X.D., ZHANG W., LAI S.K..: On traveling wave modes of axially moving string and beam. *Shock & Vibration* 2019, ID 9496180

[4] RI K., HAN P., KAN I.: KIM V.: CHA H.: Nonlinear forced vibration analysis of composite beam combined with DQFEM and IHB. *AIP Advances* 2020, **10**:085112.

[5] MARINCA V, HERISANU N, MARINCA B: *Optimal Auxiliary Functions Method for Nonlinear Dynamical Systems.* Springer: Berlin, 2021.