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Method of inversion of Laplace transform in some problems of dynamic elasticity

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Abstract: the approach for analytical inversion of Laplace transform is proposed for some widely used in dynamic elasticity theory problems cases. The theorems are formulated and proved in the general form, and the proves for the series converges are done for two important cases, which frequently arise in applications.

Keywords: Laplace transform, analytical inversion, series

1. Introduction

Laplace transform is widely used in many applications. It is worth noting dynamic problems of elasticity, such as Lamb problem, problem for elastic semi-strip etc. The most difficult part is in inversion of Laplace transform [1]. As it is known, numerical inversion of Laplace transform is not correct problem, so the search for new formulae for analytical inversion of Laplace transform is extremely relevant. In this work, the new approach for the analytical inversion of Laplace transform of the form that is widely used in many problems is proposed.

2. Results and Discussion

The following transform is considered

$$1/\left(c_0+\sum_{i=1}^{N} c_i e^{-w_i(s)}\right) \tag{1}$$

Here $w_i(s), i=\overline{1,N}$ are continuous functions, $w_i(s)>0, i=\overline{1,N}, \operatorname{Re} s>0$, $c_i, i=\overline{1,N}$ and $c_0 \neq 0$ are real constants or functions that do not depend on s, $N \geq 1$ is natural number.

In the case when $w_i(s)=n_i w(s), n_i \in \mathbb{N}, i=\overline{1,N}$ the theorem 1 can be used.

Theorem 1. If $L^{-1}\left[e^{-kw(s)}\right]$ exsists for each $k=0,1,2,\ldots$ and the series $\sum_{k=0}^{\infty}\frac{f^{(k)}(0)}{k!}L^{-1}\left[e^{-kw(s)}\right]$ converges, than the following formula takes place

$$L^{-1}\left[1/\left(c_0+\sum_{i=1}^{N} c_i e^{-n_i w(s)}\right)\right]=\sum_{k=0}^{\infty}\frac{f^{(k)}(0)}{k!}L^{-1}\left[e^{-kw(s)}\right] \tag{2}$$

Here $f\left(z\right)=1/\left(c_0+\sum_{k=1}^{N}c_k z^{n_k}\right)$.

In the most general case when $w_i\left(s\right)=\sum_{j=1}^{m}n_{ij}w_{q_j}\left(s\right), n_{ij}\in\mathbb{N}, i=\overline{1,N}, j=\overline{1,m}$ for some fixed numbers $1\le q_j\le N, j=\overline{1,m}$ the theorem 2 can be used.

Theorem 2. If $L^{-1}\left[e^{-k_j w_{q_j}(s)}\right], j=\overline{1,m}$ exsists for each $k_j=0,1,2,...$ and the series $\sum_{k_1,...k_m=0}^{\infty}\frac{1}{k_1!...k_m!}\frac{\partial^{k_1+...+k_m}f\left(0,...,0\right)}{\partial z_1^{k_1}...\partial z_m^{k_m}}L^{-1}\left[e^{-k_1 w_{q_1}(s)}\right]*...*L^{-1}\left[e^{-k_m w_{q_m}(s)}\right]$ converges, than the following formula takes place

$$L^{-1}\left[1/\left(c_0+\sum_{i=1}^{N}c_i e^{-\sum_{j=1}^{m}n_{ij}w_{q_j}(s)}\right)\right]=\sum_{k_1,...k_m=0}^{\infty}\frac{1}{k_1!...k_m!}\frac{\partial^{k_1+...+k_m}f\left(0,...,0\right)}{\partial z_1^{k_1}...\partial z_m^{k_m}}L^{-1}\left[e^{-k_1 w_{q_1}(s)}\right]*...*L^{-1}\left[e^{-k_m w_{q_m}(s)}\right] \quad (3)$$

Here $f\left(z_1,...,z_m\right)=1/\left(c_0+\sum_{k=1}^{N}c_k\prod_{j=1}^{m}z_j^{n_{kj}}\right)$.

The most difficulty in using of these two theorems is in the proving that the series in right-hand sides in (2) and (3) converges. Such proves were done for two most frequent cases of the functions $w\left(s\right)$ and $w_i\left(s\right), i=\overline{1,N}$:

- $w(s)=As$ or $w(s)=b\sqrt{s^2+a^2}$;
- $w_i(s)=s\sum_{j=1}^{m}n_{ij}A_{q_j}, i=\overline{1,N}$ or $w_i(s)=\sum_{j=1}^{m}n_{ij}b_j\sqrt{s^2+a_j^2}, i=\overline{1,N}$.

Here $A, b, A_{q_j}, b_j>0, a, a_j, j=\overline{1,m}$ are some real constants or functions that do not depend on *s*, $q_j, j=\overline{1,m}$ are natural numbers.

3. Concluding Remarks

The new method that allows to derive analytical inversion of Laplace transform for some cases is proposed. The theorems are formulated and proved in general form, and two frequently used cases are considered, for which the series convergence were proved. These formulae can be used in many applied problems, such as dynamic elasticity problem for a semi-strip.

References

[1] DOETSCH G: *Introduction to the Theory and Application of the Laplace Transformation*. Springer-Verlag: New York, 1974