

The response of nonlinear dynamic systems via Wavelet-Galerkin method in the time-frequency domain

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Abstract: The periodic generalized harmonic wavelet (PGHW) method is used to analyze the response of nonlinear dynamic systems with seismic excitation. This excitation is a non-stationary stochastic process with non-uniform modulation. The nonlinear term of the system is a typical cubic nonlinear term. First, the theoretical background of PGHW is briefly introduced, and the relationship between the power spectral density (PSD) of the stochastic process and the corresponding wavelet coefficients is given. Then, a set of algebraic equations for solving the wavelet coefficients can be obtained by using the Wavelet-Galerkin method. The Quasi-Newton method is used to solve these equations. It is more efficient than the Newton method because it does not require the calculation of complex Jacobian matrices. Then, the displacement and estimated power spectral density (EPSD) of the response can be obtained by using the wavelet coefficients. In the numerical example, the ode45 and Monte Carlo methods are used to verify the correctness of the results.

Keywords: nonlinear dynamic system, stochastic process, wavelet method, Quasi-Newton method, frequency domain

1. Introduction

The wavelet analysis is first proposed in 1980s [1]. Based on wavelet theory, harmonic wavelet (HW) [2], generalized harmonic wavelet (GHW) [3], periodic generalized harmonic wavelet (PGHW) [4] are proposed. The PGHW is a good method to deal with the vibration signals with limited duration in the engineering field. The main composition of Fourier transform is sinusoids of different frequencies, and the main composition of wavelet transform is wavelets of different scales and positions.

2. Results and Discussion

The Wavelet-Galerkin method is used to deal with the dynamic equation of the nonlinear system. Then, a set of algebraic equations for solving the wavelet coefficients can be obtained. The Quasi-Newton method is used to solve these equations. It approximates the Jacobian matrix by using the difference quotient, DFP and BFGS algorithms which simplifies the calculation process. Then, the displacement of response $x(t)$ can be obtained and shown in Fig. 1.

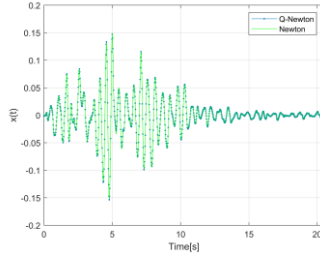


Fig. 1. The displacement of $x(t)$

From Fig. 1 we can see that the results by using Newton and Quasi-Newton methods are consistent. But the total time required for the Newton method is 327s, for the Quasi-Newton method 67s. Thus, the Quasi-Newton method is five times faster than the Newton method.

For the stochastic excitations (Nsample=200), the surface and contour of the response EPSD can be obtained and shown in Fig. 2. They all show the consistency of the results.

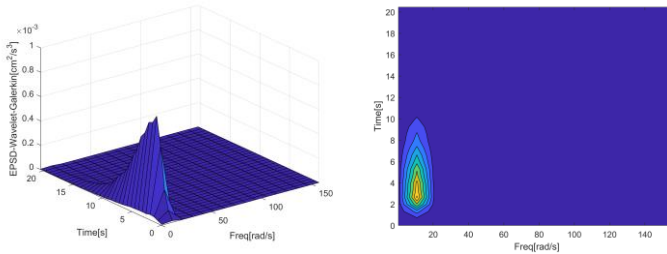


Fig. 2. The surface and contour of the response EPSD

3. Concluding Remarks

The displacement and EPSD of the responses can be obtained by using the PGHW. The Quasi-Newton method is used to solve those algebraic equations, which is more efficient than the Newton method.

References

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