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New Non-Stationary Solutions of the Restricted Three-Body Problem

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Abstract: The classical restricted three-body problem is investigated analytically on basis of the concept of forces center. In the introduced special non-inertial central reference frame, we obtained new differential equations of the classical restricted three-body problem. An invariant of the forces center have been established in this reference frame. The problem is divided into two subproblems – the triangular and the collinear restricted three-body problems. New analytical exact particular non-stationary solutions of these two subproblems are found out.

Keywords: restricted three-body problem, libration points, non-inertial central reference frame, exact particular non-stationar solutions.

1. Introduction

The motion of small natural or artificial celestial body in the gravitational field of two primary bodies is well described by the mathematical model that is known as the restricted three-body problem [1-3]. Due to absence of a general solution in finite form, much aspects of the problem are studied by various qualitative and numerical methods. Search for new exact particular analytical solutions is actual except for those which are already known. In the present paper the classical restricted three-body is investigated analytically with application of a new concept based on a notion of the forces center [3].

2. Results and Discussion

We derived the differential equations of motion in the restricted three-body problem in a new special non-inertial central reference frame with the origin located at the forces center [4]. Masses of primary bodies are different. Based on properties of the new special non-inertial central reference frame, an invariant of the forces center in the restricted three-body problem is found out in analytical form. In the introduced special non-inertial central reference frame, the restricted three-body problem is divided into two different subproblems on the level of differential equations of motion [4,5]. The first one is the triangular restricted three-body problem when three bodies form triangle during all the time of their motion. The second one is the collinear restricted three-body problem when three bodies lie on the same straight line during all the time of their motion. A possibility of such separation of investigation in special non-inertial reference frame is provided by the invariant of the forces center obtained in this reference frame.

We obtained new exact analytical particular non-stationary planar solutions of differential equations of motion in the *triangular circular restricted three-body problem* in the form of isosceles triangle with variable height in the special non-inertial central reference frame [5]. The corresponding stationary solutions are well-known as the Lagrange triangular libration points. Note that obtained solutions coincide with the corresponding formulas in the particular case of the symmetrical problem of two fixed

centers [2]. We also obtained the differential equations of motion of the triangular restricted three-body problem in the rotating special non-inertial central reference frame in the pulsating variables and the obtained solutions of the triangular problem are analyzed.

There have been derived differential equations of the collinear restricted three-body problem in the rotating non-inertial central refernce frame in pulsating variables [6]. We obtained three new differential equations of motion in the collinear restricted three-body problem, in three areas of possible location of the massless body, sationary solution of which corresponds to three well-known Euler libration points. We established new exact non-stationary particular analytical solutions of obtained three new differential equations of motion of the *circular collinear restricted three-body problem*.

3. Concluding Remarks

There have been obtained new exact particular analytical solutions of the planar circular restricted three-body problem. These results generalize the well-known two stationary Lagrange solutions and three stationary Euler solutions onto new areas in form of non-stationary solutions.

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