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The Dynamical Behaviour of a Quantum Impact Oscillator

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Abstract: It is known that an impact oscillator exhibits complex dynamics and chaos close to the grazing bifurcation. What happens to a similar system in the quantum domain? To explore this question, we simulate the quantum versions of a hard-impacting system and show that such systems would exhibit complex non-chaotic behaviour with a countable infinite number of frequencies.

Keywords: Impact oscillator, quantum mechanics, chaos

1. Introduction

In this work, we explore the dynamics of an impact oscillator in the quantum domain. The classical analogue comprises a mass-spring system that can impact a wall placed at a distance. Such a system is known to have square root singularity and exhibits chaos [1,2]. In the quantum version, the potential function is similar to that of a harmonic oscillator but assumes an infinite value at the position of the wall (Fig.1(a)).

We solve the Schrödinger equation for these systems numerically to obtain the dynamics of the wavefunction. To obtain the visual analogue of the phase space dynamics, we use the Wigner function. The dynamics are finally analysed by obtaining the overlap integral with the initial wavefunction to obtain a real-valued time series. Analysis of the time series shows aperiodic dynamics but without any sensitive dependence on the initial condition.

2. Results and Discussion

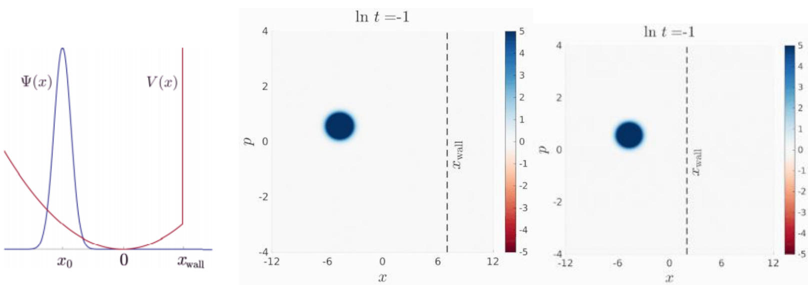

Fig. 1. The potential function and the initial wavefunction (a), and the evolution of the Wigner function: when the wall is placed away from the grazing condition (b) and when the classical oscillator undergoes impacts (c).

Fig.1(a) shows the potential function of the system and the initial wavefunction, which is a shifted Gaussian function—the quantum analogue of the mass being released at a stretched position of the spring. We also show the dynamics of the Wigner function in the phase space under two different conditions. Fig.1(b) shows it under the condition when the classical oscillator makes no contact with the wall, and Fig.1(c) shows it under impacting condition. Note that a quantum system's wavefunction is spread over a range of the phase space, and so the existence of the wall influences the dynamics even when the wall is placed away from the classical impacting condition.

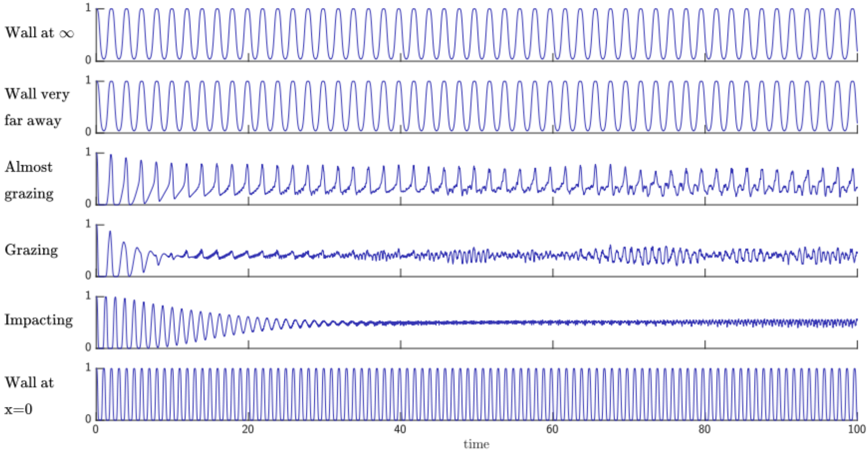

Fig. 2. The time series obtained by computing the overlap integral for different wall positions.

Fig.2 shows that the system exhibits periodic behaviour when the wall is placed far away from the oscillator and when it is placed exactly at the unstretched position of the spring. The system exhibits aperiodic orbits for intermediate positions of the wall. We have analysed these orbits and have found that the Lyapunov exponent is zero, implying no sensitive dependence on the initial condition. The Fourier spectrum reveals an infinite number of discrete frequency components.

3. Concluding Remarks

This work reveals that quantum systems may exhibit a new type of dynamics, where the orbit is aperiodic, but the spectrum is neither spread over all frequencies like a chaotic system nor does it have a finite number of frequency components like quasi-periodic dynamics. It has an infinity of discrete frequency components.

References

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