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Normal Form on nonlinear systems and Gröbner based exploitation of resonances

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Abstract: The Normal Form application is introduced on dynamical systems with diagonal and Jordan block matrices. Spectrum rearrangement allows to exhibit resonant terms associated with periodic behaviours of the system. Resonance equations are forming Gröbner generators and associated Gröbner bases allow to linearize equations depending on amplitude-frequency conditions.

Keywords: Normal Form, Gröbner bases, nonlinear, spectrum

1. Introduction

This work focuses on the study of general nonlinear dynamics systems using the Normal Form perturbation method [1,2] as long as the amplitude of the response is relatively limited [3]. The Gröbner bases are then used to solve algebraic equations of normal coordinates. After spectrum optimisation, nonlinear terms can be classified between non-resonant and resonant terms providing new physical equations. Gröbner bases firstly presented by Buchberger [4] are generalising the polynomial division of a polynomial by other multivariable polynomial. Normal Form compatibility equations are used to generate the ideals and then to reduce governing Normal coordinates equations by the generalised Euclidian division. In optimal cases, linear solutions $X = M(\Omega).U$ are obtained, supposing that potentially nonlinear generators equations are verified. Except the spectrum definition, this process can be automatized [6,7].

2. Results and Discussion

2.1. *General case*

Let us consider following general dynamical system:

$$\dot{X} = A.X + F(X) \tag{1}$$

where $F(X) = F_2(X) + \cdots + F_n(X)$ are vectors of polynomial nonlinearities. Let's assume that $A = P^{-1}.D.P$, with $D = D_0 + D_1(\epsilon)$ a diagonal matrix, $\mathcal{O}\big(D_1(\epsilon)\big) \geq 1$. Normal coordonates $Y = P^{-1}.X = U + \phi_2(U) + \cdots + \phi_n(U)$ allow to introduce normal transformation $\phi = \phi_2 + \cdots + \phi_n$ and resonant terms $R = R_2 + \cdots + R_n$. The system takes the new form

$$\begin{cases} \dot{Y} = D_0.Y + D_1.Y + P^{-1}.F(P.Y) \\ \dot{D}_1 = 0, \quad \mathcal{O}(D_1.Y) = 2, \quad \mathcal{O}\big(P^{-1}.F(P.Y)\big) \geq 3 \end{cases} \tag{2}$$

The Normal Form resolution is done degree per degree and leads to the expression of X as a function of U and to compatibility equations:

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$$\begin{cases} X = P^{-1}.Y = P^{-1}.(U + \phi_2(U) + \cdots + \phi_n(U)) \\ \dot{U} - D_0.U - R_2 - \cdots - R_n = 0 \end{cases} \quad (3)$$

Compatibility equations are providing amplitude-frequency conditions in order to assure periodic behaviours. Euclidian division by the Gröbner bases elements is applied to $X(U)$. Gröbner generators are defined by these equations as $G_k = 0$ and allow to decompose $X = M_1(\Omega).U + M_2(U).U$, where $M_2(U)$ is a simplified expression derived from normal transform and Gröbner bases.

2.2. *An example: a two Degrees of Freedom (DoF)*

To illustrate this process, we apply it to a two DoF cubic nonlinear system where $D = D_0$. Normal form equations are

$$\begin{cases} \deg_1: DU = DU \\ \deg_2: \partial\phi_2 DU - D\phi_2(U) = H_2 - R_2 \\ \deg_3: \partial\phi_3 DU - D\phi_3(U) = H_3 - R_3 \\ G_k = i\Omega u_k - \lambda_k u_k - R_{2k} - R_{3k} \end{cases} \quad (4)$$

where H_k gathers k-order nonlinear terms. We obtain the final normal expressions

$$\begin{cases} X = M_1(\Omega).U + M_2(U).U \\ G_1, \ldots, G_4 = 0 \end{cases} \quad (5)$$

For this system, M_1 could be associated to linear homogenous solution if we obtain $M_2 = 0$. All nonlinearities remain in the variety of $G_1, \ldots., G_4$. In this two DoF case, $M_2(U)$ terms are either $u_1 u_4$ or $u_2 u_3$ terms. According to compatibility equations, the variety and accepted amplitudes impose that either $u_1 = 0$ or $u_3 = 0$. In both cases, $M_2 = 0$ and a linear expression is finally obtained.

3. Concluding Remarks

The solution of the system cannot be expressed linearly in Normal coordonates in general, unless uncoupling issuing from the analysis of the frequency compatibility condition due to the amplitude equations. Nevertheless, a perspective work could be that Gröbner bases should be also used in order to define new normal coordinates and introduce less pairing, especially when D is a Jordan matrix [5].

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