2021 DSTA dys-ta.com
16th INTERNATIONAL CONFERENCE
Dynamical Systems – Theory and Applications
December 6-9, 2021, On-line

Data-driven reduced-order nonlinear models from spectral submanifolds

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Abstract: Data-driven model reduction methods are widespread for linear dynamical systems, while available approaches for nonlinear systems tend to be sensitive to parameter changes and offer limited prediction potential outside the range of data used in their construction. With this contribution, we present an approach that extracts explicit nonlinear models from data capitalizing on the theory of spectral submanifolds. Without specific assumptions on the type of observables or the kind of measurements, our method identifies nonlinear models that uncovers geometric nonlinearities and nonlinear damping in the observed dynamics. Our reduced-order models, which are trained on unforced trajectory data, also show great accuracy in predicting forced-responses of the nonlinear dynamical system. We validate our algorithm in several examples that feature synthetic or experimental data from structural vibrations or fluid dynamics.

Keywords: nonlinear oscillations, normal forms, invariant manifolds, machine learning

1. Introduction

Date-driven modeling is often coupled with dimensionality reduction for generating computationally efficient and possibly interpretable dynamical models. The most common approaches in the literature are Principal Orthogonal Decompositions (POD) followed by Galerkin projections [1] or Dynamic Mode Decomposition (DMD) [2]. While the former requires the knowledge of the full vector field generating the dynamics, the latter is purely data-driven. Yet DMD is only efficient when one uses advantageous observables and the system has a single steady state near which a linear approximation to the dynamics is feasible [3]. Machine learning approaches based on POD are also available [4] but tend to be sensitive and have limited potential for extrapolation and prediction.

We present here an approach based on the theory on spectral submanifolds (or SSMs, for short) which are the unique, smoothest invariant manifolds that act as nonlinear continuations of the modal subspaces of the linearized system [5]. Reducing the dynamics to these SSMs enables us to extract reduced-order models from generic observables, addressing most of the issues we have mentioned for available methods. We illustrate this approach with a numerical example coming from structural dynamics, which shows how our model trained on transient data is capable of extracting information on the system and of predicting forced responses.

2. Results and Discussion

We consider a straight, clamped-clamped Von Kármán beam [6], shown in Fig. 1(a), made of aluminum and has length 1 [m] and thickness 1 [mm]. For the numerical simulations, we use a finite element model with 16 elements (45 degrees of freedom). We observe trajectories of the beam midpoint that are initialized on the static deflection occurring from loading the midpoint, cf. Fig. 1(a,b). The spectral gap among the linearized system eigenvalue is such that these trajectories rapidly converge on the slowest two-dimensional SSM, on which they decay toward the equilibrium. We feed our algorithm with these scalar signals and seek learn this two-dimensional SSM. After embedding the trajectory in a suitable space, a manifold parametrization is identified, as well as the normal form model for the dynamics. This latter is set to be of $O(7)$ and it has the form

$$\begin{aligned}\dot{\rho} &= -c(\rho)\rho \\ \dot{\theta} &= \omega(\rho)\end{aligned} \tag{1}$$

where c describes the nonlinear damping, while ω is instantaneous frequency of decaying oscillations. The variations of these amplitude-dependent properties are shown in Fig. 1(c,d). We have used one trajectory for training which our reduced-order model can reconstruct with less than 2% relative room mean squared error. Figure 1(e) shows that our model is also able to predict forced responses for several forcing values, when compared to the analytical results obtained from SSMtool [7] and to direct numerical integrations. The external forcing is applied on the FEM model at the midpoint, while the forcing amplitude for the reduced-order model is calibrated on the sweep having the lowest amplitude.

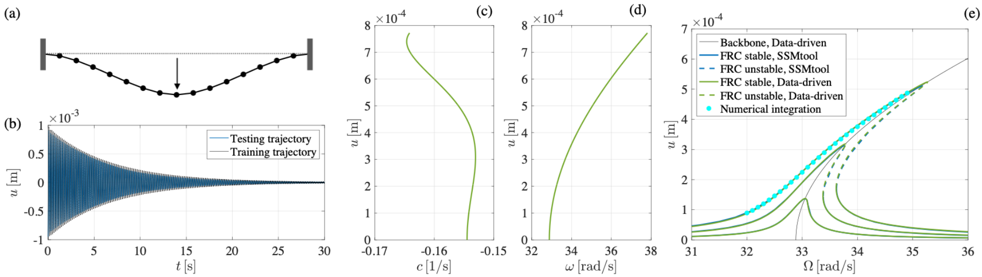

Fig. 1. Plot (a) depicts the Von Kármán beam model, while (b) shows the trajectories of the midpoint. Damping and frequency as function of the amplitude are illustrated in plots (c,d), while plot (e) compares the forced response curves (FRCs) for different forcing levels obtained with the data-driven model, analytical computations and numerical integration.

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