

Maximal attractor range of multiscroll chaotic attractors: classification of symmetries in hidden bifurcation routes

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Abstract: Multiscroll chaotic attractors of a 3D autonomous Chua-like system with saturated function series are considered. The method used to reveal hidden bifurcation routes (HBR) depending upon two parameters p and q is similar to the method recently introduced for classical Chua multiscroll attractors. However, the HBR of the saturated function series system are characterized by the maximal range extension (MARE) of their attractors and the appearance order of the scrolls which presents interesting symmetries with respect to both parameters. The approximate value of MAREs depending upon both parameters are given. They are linked to the size of the scrolls. This allows to obtain the coding of the HBR without any numerical computation.

Keywords: Hidden bifurcation, Multiscroll chaotic attractor, Saturated function series, Symmetry.

1. Introduction

Several methods have been proposed in the last three decades to generate multiscroll chaotic attractors due to their promising applications in various real-world technologies, like piecewise linear functions and nonlinear modulating functions. In electronic circuits, step, hysteresis, and saturated circuits have been built for generating multidirectional multiscroll chaotic attractors. For all the multiscrolls already known, the number of scrolls (or spirals) is a fixed integer, which depends on one or more discrete parameters. Although the majority of such multiscroll generations are known for many years, it is only recently that they are studied under the scope of bifurcation theory. Menacer et al. [1] changed the paradigm of discrete parameters by introducing hidden bifurcations, generating multiscrolls in a family of systems possessing a continuous bifurcation parameter. Then, all the classical theories of dynamical systems and their powerful tools can be used for studying the multiscrolls. In this article, the focus is on the study of the symmetries of hidden bifurcation routes in the multiscroll chaotic attractors generated by saturated function series [2].

2. Results and Discussion

The following example of multiscroll chaotic attractor (1-2) is considered:

$$\begin{cases} \dot{x} = y \\ \dot{y} = z \\ \dot{z} = -ax - by - cz + d_1 f(x, k, h, p, q) \end{cases} \quad (1)$$

with parameter values $a = b = c = d_1 = 0.7$. The number n of scrolls (also denominated spirals in [1]) satisfies $n = p + q + 2$.

$$f(x, k, h, p, q) = \begin{cases} y_{1,k} & \text{if } x > qh + 1 \\ y_{2,k,i} & \text{if } |x - ih| \leq 1, \quad i \in [-p, q] \\ y_{3,k,i} & \text{if } l_{1,i} \leq x \leq l_{2,i}, \quad i \in [-p, q - 1] \\ y_{4,k} & \text{if } x < -ph - 1 \end{cases} \quad (2)$$

Using the method defined in [1], bifurcation routes versus a continuous parameter ε in which the number of spirals is increasing are revealed (Fig. 1). The maximal attractor range extension (MARE p,q) is the size of the x -projection of the considered attractor defined by parameter values p and q , when $\varepsilon=1$ and $t \rightarrow +\infty$. It is shown that MARE $p,q = [-20 \times (p+1), 20 \times (q+1)]$, and bifurcation routes can be coded using a basic cell ($[0, 20]^{(\varepsilon)} / L$) or ($[-20, 0]^{(\varepsilon)} / R$) and some symmetry rules.

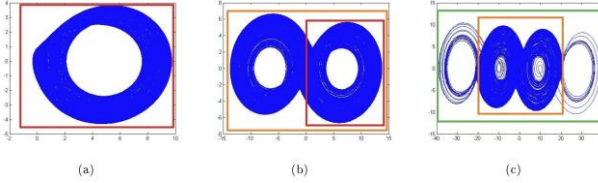


Fig. 1. The increasing number of spirals of system (1-2) modified following [1] according to increasing ε values, when $p = 2$ and $q = 3$, $k = 10$ and $h = 20$. (a): The first scroll between lies between 0 and 20 for $\varepsilon = 0.42$, (b): The second scroll on the left for $\varepsilon = 0.6$, (c): The third and the fourth scrolls: two left-right symmetrical for $\varepsilon = 0.95$. The horizontal axis is the x -axis, the vertical axis is the y -axis.

$p \backslash q$	0	1	2	3
0		[-40,20]/ B/L	[-60,20]/ B/L/L	[-80,20]/ B/L/L/L
1	[-20,40]/ B/R	[-40,40]/ B/2Sym	[-60,40]/ B/2Sym/L	[-80,40]/ B/2Sym/L/L
2	[-20,60]/ B/R/R	[-40,60]/ B/2Sym/R	[-60,60]/ B/2Sym/2Sym	[-80,60]/ B/2Sym/2Sym/L
3	[-20,80]/ B/R/R/R	[-40,80]/ B/2Sym/R/R	[-60,80]/ B/2Sym/2Sym/R	[-80,80]/ B/2Sym ³

Fig. 2. Symmetries of the hidden bifurcation routes : HBR p,q and MARE p,q , numerically computed (black) and inferred from symmetry rules (red).

3. Concluding Remarks

Hidden bifurcation routes in multiscroll chaotic attractors generated by saturated function series have been explored. These routes have interesting symmetries with respect to the two parameters allowing to obtain their coding using the new introduced tool (MARE p,q) without any numerical computation.

References

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