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Waves in a beam resting on a bilinear Winkler foundation

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Abstract: The work addresses the problem of propagation of periodic waves in a beam resting on a bilinear substrate, having different soil stiffness in "compression" and "tension". The closed form solution is obtained, and some aspects of the complex behaviour are illustrated.

Keywords: wave propagation, bilinear foundation, Euler-Bernoulli beam, phase velocity

1. Introduction

The problem of a beam resting on a bilinear elastic foundation, i.e. a Winkler soil with different stiffnesses in "compression" and in "tension", has been scarcely studied in the past, despite its practical relevance and despite the fact that it is one of the few nonlinear problems for which an exact mathematical solution can be obtained.

In the static case it has been addressed in [1] and, recently, in [2]. In the more interesting dynamical case, it has been initially studied in [3] where an analytical solution is obtained by means of a perturbation approach. The case of a load moving at a constant velocity have been considered in [4] and [5] by means of a numerical approach. Much more investigated has been the case of a unilateral soil, which is a particular case of a bilinear foundation when one stiffness is set to zero, see [5,6] for an interesting literature survey.

In previous works the problem of a free wave propagation has not been considered, and this constitutes the goal of this work, where periodic waves are studied. This allows us to obtain an analytical solution, and to highlight the complex nature of the response due to its (piecewise) nonlinearity.

2. The problem and main results

The governing equation for the undamped unforced Eulr-Bernoulli beam resting on a bilinear Winkler foundation is

$$\rho A\ddot{u} + EJu^{IV} + f(u)\, u = 0, \tag{1}$$

where u is the transversal displacement, ρA the mass per unit length, EJ the bending stiffness, and

$$f(u) = \hat{h}_1 \text{ if } u < 0;\ \hat{h}_2 \text{ elsewhere.} \tag{2}$$

Equation (2) underlines the bilinearity of the Winkler foundation, which is the unique source of nonlinearity. The solution is sought after in the travelling wave form $u(x,t)=U(s)$, $s = x - \hat{c}t$, where $\hat{c}$ is the phase velocity, to be determined. Defining the dimensionless parameters (L is the wavelength)

$$c = \hat{c}\, L\, (\rho A/EJ)^{1/2}, \quad h_{1,2} = \hat{h}_{1,2}\, L^4/EJ, \quad \xi = s/L, \tag{3}$$

we obtain that $U(s)$ satisfies

$$U^{IV} + c^2\, U^{II} + f(U)\, U = 0. \tag{4}$$

Looking for a periodic wave we introduced the following boundary conditions guaranteeing the continuity of the displacements, rotations, bending moments and shear forces:

$$U(0) = U(\alpha) = \Delta U(0) = \Delta U(\alpha) = \Delta U^{I}(0) = \Delta U^{I}(\alpha) = \Delta U^{II}(0) = \Delta U^{II}(\alpha) = \Delta U^{III}(0) = \Delta U^{III}(\alpha) = 0. \quad (5)$$

In (5) $\alpha \in [0,1]$ is the parameter that divides the "compression" ($U<0$, $\xi \in [0, \alpha]$) and "tension" ($U>0$, $\xi \in [\alpha, 1]$) parts of the domain.

Equations (4) and (5) constitutes a nonlinear eigenvalue problem, where the unknowns are $c(h_1,h_2)$ and $\alpha(h_1,h_2)$, together with the modal displacement $U(\xi)$. Note that in the linear case $h_1 = h_2 = h$ the solution is given by $\alpha=1/2$ and $c_{linear}(h) = 2\pi\ (1 + h/(2\pi)^4)^{1/2}$. For the bilinear case $h_1 \neq h_2$ the solution is reported in Fig. 1.

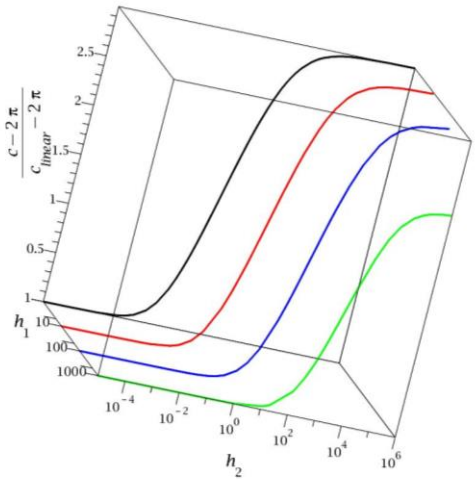

Fig. 1. The solution $c(h_1,h_2)$.

3. Concluding remarks

The propagation of a periodic wave in a bilinear elastic foundation has been considered. The exact close form solution has been obtained thanks to the piecewise linearity of the problem.

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