DSTA 2021
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16th INTERNATIONAL CONFERENCE
Dynamical Systems – Theory and Applications
December 6-9, 2021, On-line

Dynamics analysis of the spatial mechanism with imperfections in the fifth-class kinematic pairs

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Abstract: The paper presents the mathematical model of the spatial mechanism with imperfect revolute and prismatic joints. Imperfections are taken into account in the form of clearance and friction in joints. Contribution to the paper includes new models of the revolute and prismatic joints formulated for the system which the kinematics is modeled using joint coordinates and homogeneous transformations. Different approaches to modeling clearance in joints, in the cut-joint and other joints of the mechanisms, will be presented in the paper. Thanks to the use of joint coordinates for joints that are not cut-joints, displacements resulting from the clearance in a joint can be taken directly from the generalized coordinate vector without the need for additional calculations. Numerical simulations allow us to analyze an interaction between links' flexibility and imperfections due to clearance and friction in the joints.

Keywords: spatial mechanism, clearance, friction, link's flexibility

1. Introduction

Different imperfections and nonlinearities in joints of mechanisms have a significant impact on their dynamics. They can cause additional vibrations, joint wear, significant reduction of accuracy and energy efficiency. Additional impact forces due to clearance can damage mechanical parts. In the literature, it can be found many papers dealing with modeling of the clearance in joints for mechanisms whose movement is described by absolute coordinates [1, 2]. Contribution to the paper includes new models of the revolute and prismatic joints formulated for RPSUP mechanism which the kinematics is modeled using joint coordinates and homogeneous transformations (Fig. 1). The presented models are a continuation of the work, the results of which are shown in [3]. In the paper, the clearance is analyzed not only in the cut-joint but also in other joints. It is assumed that the clearance exists at cut-joint *C* and slider (2,1). The dynamics equations of motion are derived using the Lagrange equations of the second kind [4]. The normal contact force is modeled using the Nikravesh-Lankarani hypothesis [5]. Friction in joints is modeled by means of the LuGre friction model [6]. It is assumed that the coupler can be flexible and the Rigid Finite Element Method is used to model effects due to the link's flexibility [4].

2. Mathematical model of a RPSUP spatial mechanism

The cut-joint technique is applied to divide the closed-loop chain into two open-loop kinematic chains. The generalized coordinate vector is composed of the following components

$$\mathbf{q} = \left[\psi^{(1,1)} \quad x^{(1,2)} \quad \psi^{(1,3,0)} \quad \theta^{(1,3,0)} \quad \varphi^{(1,3,0)} \quad \mathbf{q}_f^{(1,3)^T} \;\vdots\; z^{(2,1)} \quad \mathbf{q}_c^{(2,1)^T} \quad \psi^{(2,2)} \right]^T \tag{1}$$

where: $\mathbf{q}_f^{(1,3)}$ - vector containing the generalized coordinates of the rigid finite elements,

$\mathbf{q}_c^{(2,1)} = \left[x^{(1,2)} \quad y^{(1,2)} \quad \psi^{(1,2)} \quad \theta^{(1,2)} \quad \varphi^{(1,2)} \right]^T$ - vector containing displacements of slider (2,1) due to the clearance in joint. It can be noted that displacements resulting from the clearance in a joint can be taken directly from the generalized coordinate vector without the need for additional calculations.

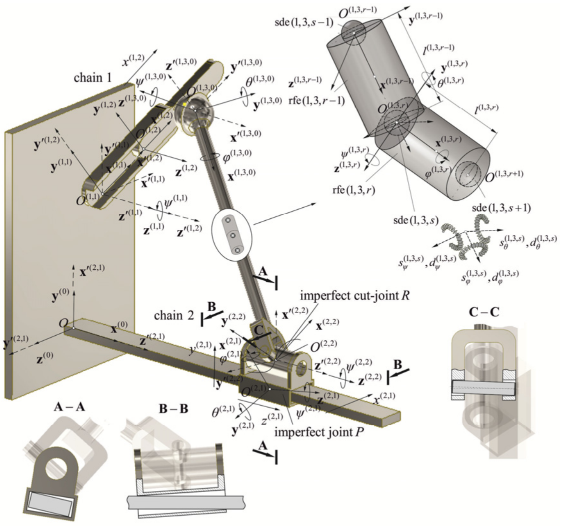

Fig. 1. Model of a rigid-flexible link spatial mechanism

Finally, dynamics equations of motion together with the state equations formulated for the LuGre friction model form a set of ordinary differential equations in the following form [3]

$$\dot{\mathbf{z}} = \mathbf{LG}\left(t, \dot{\mathbf{q}}, \mathbf{z}\right) \tag{2.1}$$

$$\mathbf{M}\ddot{\mathbf{q}} = \mathbf{Q} - \mathbf{h} - \mathbf{g} + \mathbf{f}_{cl} - \mathbf{f}_{flex} - \mathbf{f}_{fric} \tag{2.2}$$

where: $\mathbf{z}$ - vector of state variables, $\mathbf{M}$ – mass matrix, $\mathbf{Q}$ - vector of external forces, $\mathbf{h}$ – vector of the Coriolis, gyroscopic and centrifugal forces, $\mathbf{g}$ – vector of gravity forces, $\mathbf{f}_{cl}$, $\mathbf{f}_{fric}$, $\mathbf{f}_{flex}$ - vector of forces resulting from clearance and friction in joints and coupler's flexibility, $\mathbf{LG}\left(t, \dot{\mathbf{q}}, \mathbf{z}\right)$ - functions of right-hand sides of the LuGre friction model. In numerical simulations interactions between the clearance in joints and coupler's flexibility will be analyzed.

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