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The influence of the load modeling methods on dynamics of a mobile crane

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Abstract: A mathematical model for the dynamics analysis of a mobile crane is presented in the paper. The proposed model of a mobile crane is in the form of a tree structure of a kinematic chain with closed-loop subchains. The carried load is treated in the two cases, as a lumped mass and a rigid body. The formulated model takes also into account the flexibility of supports, tires, rope(s), and drives. Dry friction in joints is also considered. The formalism of joint coordinates and homogeneous transformation matrices are used to describe the kinematics of the crane. The equations of motion are derived using the Lagrange equations of the second kind. These equations are supplemented by the Lagrange multipliers and constraint equations formulated for each cut-joint.

Keywords: mobile crane, dynamics, flexibility, friction, load modeling

1. Introduction

The proposed mathematical model of a mobile crane is presented in Fig.1. In this model, a main structure of the crane is modeled in the form of open-loop kinematic chain mounted on the body of a vehicle. The hydraulic cylinders are modeled in the form of closed-loop subchains. The load is modeled in the form a lumped mass with three degrees of freedom [1,2] and a rigid body six three degrees of freedom [3,4]. The formulated model takes into account the flexibility of supports, tires, rope(s), and drives.

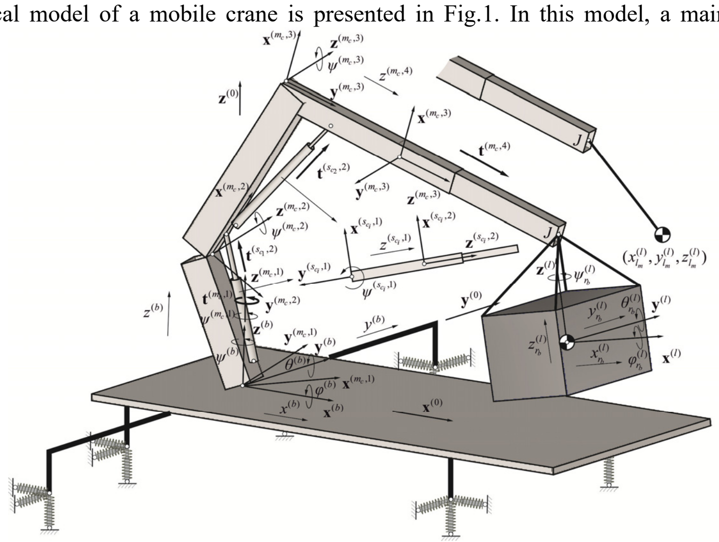

Fig. 1. Model of a mobile crane

The LuGre friction model is used to take into account dry friction in joints. The formalism of joint coordinates and homogeneous transformation matrices, based on the Denavit-Hartenberg notation, are used to describe the kinematics of the crane. Equations of motion are derived using the Lagrange equations of the second kind. These equations are supplemented by the Lagrange multipliers and constraints equations formulated for joints in which closed-loop kinematic chains are divided using the cut-joint technique.

Vector of the generalized coordinates is defined as follows

$$\mathbf{q} = \begin{bmatrix} \mathbf{q}^{(b)^T} & \mathbf{q}^{(m_c)^T} & \mathbf{q}^{(s_{c_1})^T} & \mathbf{q}^{(s_{c_2})^T} & \mathbf{q}^{(l)^T} \end{bmatrix}^T, \tag{1}$$

where: $\mathbf{q}^{(b)} = \begin{bmatrix} x^{(b)} & y^{(b)} & z^{(b)} & \psi^{(b)} & \theta^{(b)} & \varphi^{(b)} \end{bmatrix}^T$, $\mathbf{q}^{(m_c)} = \begin{bmatrix} \psi^{(m_c,1)} & \psi^{(m_c,2)} & \psi^{(m_c,3)} & z^{(m_c,4)} \end{bmatrix}^T$,
$\mathbf{q}^{(s_{c_1})} = \begin{bmatrix} \psi^{(s_{c_1},1)} & z^{(s_{c_1},2)} \end{bmatrix}^T$, $\mathbf{q}^{(s_{c_2})} = \begin{bmatrix} \psi^{(s_{c_2},1)} & z^{(s_{c_2},2)} \end{bmatrix}^T$,

$$\mathbf{q}^{(l)} = \begin{cases} \begin{bmatrix} x_{l_m}^{(l)} & y_{l_m}^{(l)} & z_{l_m}^{(l)} \end{bmatrix}^T & \text{if lumped mass,} \\ \begin{bmatrix} x_{r_b}^{(l)} & y_{r_b}^{(l)} & z_{r_b}^{(l)} & \psi_{r_b}^{(l)} & \theta_{r_b}^{(l)} & \varphi_{r_b}^{(l)} \end{bmatrix}^T & \text{if rigid body.} \end{cases}$$

The state equations for the LuGre friction model (formulated for each joint with friction) together with the dynamics' equations of motion can be written in the following general form

$$\dot{\mathbf{z}}^{(\alpha)}\Big|_{\alpha\in\{m_c,(s_{c_1}),(s_{c_2})\}} = \mathbf{LuGre}\left(t, \dot{\mathbf{q}}^{(\alpha)}, \mathbf{z}^{(\alpha)}\right), \tag{2.1}$$

$$\begin{bmatrix} \mathbf{M}(\mathbf{q}) & -\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})^T \\ \mathbf{C}(\mathbf{q},\dot{\mathbf{q}}) & \mathbf{0} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{q}} \\ \mathbf{f}_j \end{bmatrix} = \begin{bmatrix} \mathbf{e}(\mathbf{q},\dot{\mathbf{q}}) - \mathbf{s}(\mathbf{q},\dot{\mathbf{q}}) - \mathbf{d}(t,\mathbf{q},\dot{\mathbf{q}}) - \mathbf{f}(t,\mathbf{q},\dot{\mathbf{q}}) \\ \mathbf{c}(\mathbf{q},\dot{\mathbf{q}}) \end{bmatrix}, \tag{2.2}$$

where: $\mathbf{z}^{(\alpha)}$ is the vector of bristles' deflections, $\mathbf{M}(\mathbf{q})$ is the mass matrix, $\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})$ is the constraints' matrix, $\mathbf{f}_j$ is the vector of the reaction forces in the cut-joints, $\mathbf{e}(\mathbf{q},\dot{\mathbf{q}})$ is the vector of the Coriolis, gyroscopic and centrifugal forces, $\mathbf{s}(\mathbf{q},\dot{\mathbf{q}})$ is the vector of the spring and damping forces formulated for the supports and rope(s), $\mathbf{d}(t,\mathbf{q},\dot{\mathbf{q}})$ is the vector of the driving forces and torques, $\mathbf{f}(t,\mathbf{q},\dot{\mathbf{q}})$ is the vector of the friction forces and torques, $\mathbf{c}(\mathbf{q},\dot{\mathbf{q}})$ is the vector of the right side of constraints' equations.

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