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Nonlinear oscillations of a complex discrete system of rigid rods with mass particles on an elastic cantilever

KATICA (STEVANOVIĆ) HEDRIH[1, 2], ANDJELKA HEDRIH[1*]

1. Department of mechanics, Mathematical Institute of Serbian Academy of Sciences and Arts (MI SANU) Belgrade, Serbia [ORCID: 0000-0001-7598-900X]
2. Faculty of Mechanical Engineering, University of Niš, Serbia [ORCID: 0000-0002-2930-5946]
* Presenting Author

Abstract: We analyzed forced oscillations of a complex cantilever. The non-linearity of the system is introduced by a spring with nonlinear properties that oscillates in a vertical plane. The description and approximations of the system are given. The system oscillates in two orthogonal planes - horizontal and vertical with four degrees of freedom in each plane. In the horizontal plane, the system oscillates with eigen frequencies of free linear oscillations; in the vertical plane with forced nonlinear oscillations. For describing oscillatory behavior of this complex system under an external single-frequency force, influence coefficients of deflection of cantilever were used. Oscillatory behavior of this complex system in vertical plane can be described by subsystems of nonlinear differential equations that are solved using a newly introduced, generalized method of variation of constants and the method of averaging, as well as the Krilov-Bogolyubov-Mitropolyski asymptotic method of nonlinear mechanics of approximation. The presented generalized methods of constants variation, together with the averaging method, opens the possibility of studying the forced nonlinear oscillations, under the influence of external forces with different frequencies, each in the corresponding resonant range frequency interval.

Keywords: nonlinear dynamics, complex cantilever, influence coefficients of deflection, averaging method, method of constant variations

1. Introduction

Complex vertical oscillations [1] and non-linear equations of motion of L-shaped beam structures [2] were studied by other authors. We modified the previously proposed elastic cantilever model with symmetric attached rigid rods with mass particles [3] introducing nonlinearity in the form of a spring with nonlinear properties that oscillates in the vertical plane. Fig.1.

2. Model and methods

The influence coefficients of cantilever deflection are determined and introduced in nonlinear differential equations for describing forced oscillations of a discrete complex system in the vertical plane. The system of 4 nonlinear differential equations described forced nonlinear oscillations in the vertical plane. The equation describing the oscillations in the vertical plane in dynamical configuration 1 is:

$$y_i = \alpha_{i1}\left[F_{01y}\sin\Omega_y t + \left(-cy_1 - c_N y_1^3\right)\right] + \alpha_{i2}\left(-2m_2\ddot{y}_2\right) + \alpha_{i3}\left(-2m_3\ddot{y}_3\right) + \alpha_{14}\left(-m_4\ddot{y}_4\right) + \\ + \delta_{i2}\left(-2m_2\ddot{y}_2\ell_2\cos\beta_2\right) + \delta_{i3}\left(-2m_3\ddot{y}_3\ell_3\cos\beta_3\right); \qquad i = 1,2,3,4 \tag{1}$$

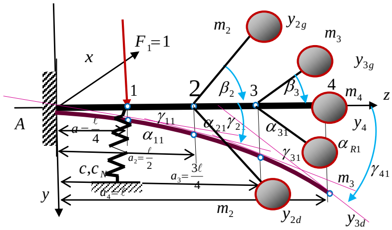

Fig. 1. Oscillations of a complex oscillatory model in the vertical plane.

The system of nonlinear differential equations is solved using a newly introduced, generalized method of variation of constants and the Krilov-Bogolyubov-Mitropolyski asymptotic method of nonlinear mechanics. The Krilov-Bogolyubov-Mitropolyski method was used for non-stationary regime when the frequency of the external force is in the resonant frequency range of one of the eigen frequencies of free oscillations of the linearized system. In this case, frequency changes with the different speed. In the method of variation of constants, the assumption is that the frequency of the external force in nonlinear system, is approximately equal to one of own eigen circular frequency [4].

3. Results and Discussion

In the case where Krilov-Bogolyubov-Mitropolyski method was used, frequency changes with the different speed. By obtained system of nonlinear differential equations along four phases and four amplitudes in the first asymptotic approximation, it is possible to obtain numerical/graphical solutions for amplitude-frequency curves of small forced nonlinear oscillations.

3. Concluding Remarks

The paper contains a generalization of the method of variation constants in combination with averaging method for obtaining a system of ordinary nonlinear differential equations of first order along corresponding number of amplitudes and phases in first approximations describing non-linear modes of the complex discrete system. This system is simpler than a system of governing differential equations along amplitudes and phases of generalized coordinates, and permits qualitative analysis of the non linear phenomena in the systems with nonlinear dynamics, specificly stability and instability of the amplitudes and phases of nonlinear modes in the first approximation.

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References

[1] GEORGIADES F, WARMINSKI J, CARTMELL P.M: Towards Linear Modal Analysis for an L-Shaped Beam: Equations of Motion. *Mechanics Research Communications* 2013, **47:**50-60.

[2] ŁYGAS K, WOLSZCZAK P, LITAK G, STACZEK P: Complex response of an oscillating vertical cantilever with clearance. *Meccanica* 2019, **54:**1689–1702.

[3] HEDRIH A, JOVANOVIC Dj, (STEVANOVIC) HEDRIH K. Theoretical modeling of forced oscillations of herbaceous plant. *XXV ICTAM,* 23-28 August 2020+1, Milano, Italy.

[4] STEVANOVIĆ K: Two-frequency non stationary forced vibrations of a beam, *Marhematical Physics,* Kiev, 1972, **12**:127-140. (in Russian).