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Nonlinear Dynamics, Stability and Control Strategies: Mathematical Modeling on the Big Data Analyses of Covid-19 in Poland

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Abstract: Detailed numerical data on covid-19 epidemic have been collected since February 2020, and now >200 countries and regions are presented in online databases. A brief review of the data analyses and mathematical modelling for different countries is given. The time series on three 'waves' of pandemic in 16 provinces of Poland are analyzed. Statistical regularities, self- and cross-correlations, common and different features in the regions are revealed. Spectral analysis of oscillating components and phase curves demonstrated non-linear quasi-regular and chaotic dynamics. Based on the comparative analyses of the 7-day averaged curves, the non-linear SEIDQRV model with time delay was estimated as the most proper mathematical model. Material parameters of the model for each 'wave' and region have been used for stability analysis and controllability of the pandemic. The $\Re$-factor (reproduction number) for each region/wave have been obtained as the stability criterion for the systems of equations of the SIR, SIRS, SEIR, SEIRS, SEIDQR models with and without time delay. Sensitivity of the models to different control functions (social restrictions, lockdown measures, availability/quality of medical treatment, vaccination level) revealed different sets of the most influencing parameters in different provinces and waves. The results are compared for similar data for Poland and other European countries. It is shown; the nonlinear dynamics and best control strategies differ at the level of the country and its regions that needs more complex local governmental measures against further development of epidemic.

Keywords: covid-19 pandemic, data analyses, nonlinear dynamics, mathematical modelling, stability and control

1. Introduction

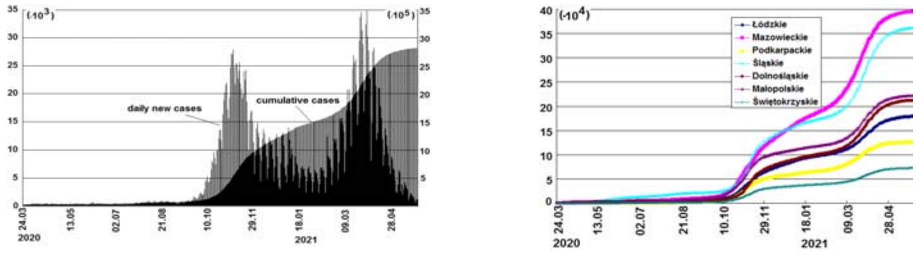
Rapid development of the covid-19 pandemic produced Big Data with high volume, velocity and variety that in combination with geographic, climatic, meteorological, social, economic data[1] give a rich source for statistical analysis and mathematical modelling [1-3]. It is a unique field of research with >60,000 papers published and indexed in ScienceDirect, ~138,000 – in PubMedCentral, >3800000 – in Google Scholar on different aspects of the pandemic development and outcomes. In this study the very first results on the detailed data analysis for Poland, its 16 provinces and neighbour European countries is presented. The most popular mathematical models of covid-19 pandemic are analyzed based on the data for each province and each of the three 'waves' of the epidemic.

2. Results and Discussion

The online databases are continuously updated by the data on new/cumulative cases, deaths, recoveries, vaccinations, and other information related to the epidemic over the world[1]. The diagrams

1 https://ourworldindata.org/ , https://www.worldometers.info/

on new daily cases $N_{nd}(t)$ in Poland confirms the current decay of the 3-rd 'wave' (Fig.1a) while similar distributions for 3 selected provinces reveal different dynamics and time delay (Fig.1b). Dynamics of the 3 'waves' in the provinces distinct (Fig.1b) and demonstrate significant differences and time delay with averaged data on the country (Fig.1a). Similar conclusions can be derived by comparison to the data treatment for USA, Canada, Australia, different European and Asian countries [3].



a b

Fig. 1. New daily and cumulative cases in Poland (a), cumulative cases in 7 provinces (b)

The most popular SEIDQR mathematical model is based on the ODEs for the numbers of susceptible (S), exposed (E), infected (I), diagnosed (D), quarantined (Q), and Recovered (R) individuals. In this study a generalization of the model for vaccinated (V) people is proposed in the form

$$\frac{d}{dt}X_j(t) = f_{j1}(X_j)X_1(t-\tau_1) + f_{j2}(X_j)X_2(t-\tau_2) + ... + f_{jn}(X_j)X_n(t-\tau_n), \qquad (1)$$

where $X_j = \{S, E, I, D, Q, R, V\}$, $f_{jn}(X_j)$ are known non-linear functions [1-3].

Simplified SIR, SIRS, SEIR, SEIRS models can be obtained from (1) by elimination of some variables and simplification the functions $f_{jn}(X_j)$. The steady state solution $\{S^0, E^0, I^0, D^0, Q^0, R^0, V^0\}$ for each of the models can be computed from (1) at $dX_j / \mathrm{dt} = 0$. Then stability of the dynamical system (1) can be estimated as the conditions $\mathrm{Re}(\lambda) < 0$ for the solutions $X_j(t) = X_j^0 + X_j^* e^{\lambda t}$. The Lyapunov exponent $\mathrm{Re}(\lambda)$ is considered as an informative parameter $\Re$ (reproduction number) for the dynamical instability of the epidemic. This critical value is important for the stability, control and prognosis local and global dynamics of the pandemic.

3. Concluding Remarks

Statistical analysis of the covid-19 Big Data for Poland, its provinces and other countries revealed non-linear quasi-regular and chaotic dynamics. The reliable estimations of the pandemic development and measures for its decay can be obtained from the proposed generalized model with time delay.

References

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