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# Bifurcations in inertial focusing of particles in curved rectangular ducts

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**Abstract:** Motion of particles suspended in a fluid flow is governed by hydrodynamic forces acting on the particle from the surrounding flow. In particular, the inertial lift force may cause particles to deviate from the streamlines of the background fluid flow. This was first demonstrated in the classical experiment of Segré & Silberberg [1] where particles suspended in flow through a straight pipe with a circular cross-section were observed to migrate to an annular region approximately 0.6 times the radius of the pipe. The phenomenon of inertial migration has found many applications in medical and industrial settings such as isolation of circulating tumour cells, separation of particles and cells, bacteria separation, cell cycle synchronization and identification of small-scale pollutants in environmental samples [2,3].

Harding et al. [4] developed a general asymptotic model for forces that govern the motion of a spherical particle suspended in a fluid flow through a curved duct and used it to investigate the inertial migration of a neutrally buoyant spherical particle suspended in flow through curved ducts with square, rectangular and trapezoidal cross-section. They identified stable and unstable equilibrium points in the cross-section of the duct which vary with the cross-section geometry, the bend radius of the duct and particle size. Moreover, they showed that for low flow rates, the lateral focusing position of particles approximately collapses with respect to a dimensionless variable $\kappa$ that is dependent on particle radius, duct height and duct bend radius.

In this talk, I will present the results of our detailed investigation of the various bifurcations that take place in a curved rectangular duct as a function of the bend radius, particle size and the aspect ratio of the rectangular cross-section. We have used the asymptotic model of Harding et al. [4] as well as a reduced order model for small particles to investigate the dynamics and bifurcations of particles. The reduced order model is a system of nonlinear coupled ODEs with a single parameter $\kappa$. This simplified model allows us to analytically investigate the stability and bifurcations. Using this reduced order model, we have been able to analytically solve for the equilibrium points and its linear stability for the limiting cases of very large and very small bend radius of the curved duct. Exploration of numerical solution of the reduced model as a function of $\kappa$ results in rich dynamical behaviour and a large variety of bifurcations. For relatively large bend radius, we observe the existence of several fixed points. These include stable nodes, unstable nodes and saddle points. As the bend radius is progressively decreased, several bifurcations take place for these fixed points such as saddle-node, pitchfork and Hopf bifurcations. At relatively small bend radius, either a stable spiral or an unstable spiral with a limit cycle remains along with one saddle point. Implications of these results for particle separation will also be discussed.

**Keywords:** Inertial microfluidics, Inertial migration, Inertial lift, Bifurcations, particle/fluid flow

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## References

[1] SEGRE G, SILBERGERG A: Radial particle displacements in Poiseuille flow of suspensions. *Nature* 1961, **189**:209-210.

[2] ZHANG J, YAN S, YUAN D, ALICI G, NGUYEN N, WARKIANI M, LI W: Fundamentals and applications of inertial microfluidics: a review. *Lab Chip* 2016, **16**:10-34.

[3] MARTEL J, TONER M: Inertial focusing in microfluidics. *Annual Review of Biomedical Engineering* 2014, **16**:371-396.

[4] HARDING B, STOKES Y, BERTOZZI A: Effect of inertial lift on a spherical particle suspended in flow through a curved duct. *Journal of Fluid Mechanics* 2019, **875**:1-43.