# Non-stationary stochastic dynamics analysis of structural systems equipped with fractional viscoelastic device

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**Keywords:** fractional operators, viscoelastic device, non-stationary white noise

## 1. Introduction

Viscoelastic materials are commonly used for vibration mitigation. The viscoelastic behaviour is defined by the Creep test, providing the Creep function. Starting from Nutting experiments (1921) [1] the creep function, C (t), is

$$C(t) = t^{\beta} / E_{\beta}\Gamma(\beta+1); \quad 0 \leq \beta \leq 1 \tag{1}$$

where C (β), β are found by best fitting procedure and Γ (·) is the Gamma function. By using the Boltzmann superposition principle for a power law creep function the fractional constitutive arise. Then, inserting a viscoelastic device into a structural system, a second order derivative, a fractional operator and elastic term appear. In this presentation the stochastic analysis for a non-stationary white noise input is discussed in time domain by using Grünwald-Letnikov integration scheme.

## 2. Results and Discussion

For a viscoelastic device with a creep function as in eq. (1), the Boltzmann superposition principle gives

$$\varepsilon(t) = \frac{1}{\mathrm{E}(\beta)\Gamma(1+\beta)} \int_0^t (t-\tau)^{\beta} \dot{\sigma}(\tau) d\tau = \frac{1}{\mathrm{E}_{\beta}\Gamma(\beta)} \int_0^t (t-\tau)^{\beta-1} \sigma(\tau) d\tau \tag{2}$$

The last equality in eq. (2) is proportional to the Riemann Liouville fractional integral, ${}_0I_t^{\beta}$ . The constitutive laws are, for quiescent system (t ≤0)

$$\varepsilon(t) = \frac{1}{E_{\beta}} \left( {}_0I_t^{\beta} \sigma \right)(t); \sigma(t) = E_{\beta} \left( {}_0D_t \varepsilon \right)(t) \tag{3.a,b}$$

where ${}_0^cD_t^{\beta}$ in eq. (4.b) is the Caputo's fractional derivative. Then, for a structural system equipped with a viscoelastic device we may write:

$$\ddot{X} + 2\zeta_\beta \omega_0 \left( {}_0^c D_t^\beta X \right)(t) + \omega_0^2 x = \Psi^{\frac{1}{2}}(t) W(t); \Psi(t) > 0 \quad (4)$$

where $\zeta_\beta$ is the (anomalous) dissipation factor, $\omega_0$ is the natural frequency, $\Psi$ (t) is a deterministic modulating function and W(t)is characterized by the correlation function $R_w(\tau) = \mathrm{E}\left[ W(t) W(t+\tau) \right] = q\delta(\tau)$, being q the strength of white noise. The operators of derivatives and integrals may be inverted to obtain

$$X + 2\zeta_\beta \omega_0 \left( {}_0 I_t^{2-\beta} x \right)(t) + \omega_0^2 \left( {}_0 I_t^2 x \right)(t) = {}_0 I_t^2 \left( \Psi^{\frac{1}{2}}(t) W(t) \right) \quad (5)$$

that, by using the Grünwald-Letnikov integration scheme [2] we get

$$\boldsymbol{A}_n \boldsymbol{X}_n = \boldsymbol{B}_n \boldsymbol{W}_n \quad (6)$$

where $\mathbf{X}_n$, $\mathbf{W}_n$ are n-vectors whose j-th component are $X(t_j)$ and $W(t_j)= \left( q\Psi(t_j)/\Delta t \right)^{1/2} N_j$, respectively. Moreover $t_j$-$t_{j-1}$= $\Delta t$; $N_j$ are realizations of standard normal random variables with E $[N_i N_j] = \delta_{ij}$ and

$$\boldsymbol{A}_n = \boldsymbol{A}_1 + 2\zeta_\beta \omega_0 \Delta t^{2-\beta} \boldsymbol{A}_{2-\beta} + \omega_0^2 \Delta t^2 \boldsymbol{A}_2; \boldsymbol{B}_n = \boldsymbol{A}_2 \Delta t^2 \quad (7)$$

being $\mathbf{A}_\gamma$ ($\gamma$=1,2–$\beta$,2) a lower bound strip matrix [2] whose first column elements are given as $A_{\gamma 1}$ =1, $A_{\gamma 2} = \gamma$, $A_{\gamma ij}$ =$A_{\gamma ij-1}$ (1+ $\gamma_{j-2}$)/(j-1) from eq. (5) we get

$$E\left[ \boldsymbol{X}_n \boldsymbol{X}_n^T \right] = \boldsymbol{A}_n^{-1} \boldsymbol{B}_n \left[ \Psi^{\frac{1}{2}}{}_n \Psi^{\frac{T}{2}}{}_n \right] \left( \boldsymbol{B}_n^{-T} \boldsymbol{A}_n^T \right) \quad (8)$$

From this equation the entire correlation matrix of the response process is easily determined.

## 3. Concluding Remarks

The stochastic analysis of a structural system with fractional viscoelastic device is presented. The forcing function is modelled as a non-stationary white noise. Integrating the eq. motion by the Grünwald-Letnikov integration scheme returns easily the time dependent statistics of response process at the discretized times $t_1, t_2, \ldots, t_n$.

## References


[1] NUTTING, P. G., J*ournal of the Franklin Institute*, Maggio 1921.

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