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On the dynamics of high-order beams with vibration absorbers

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Abstract: A novel solution procedure is proposed for a class of high-order boundary value problems governing the dynamic response of beams equipped with vibration absorbers. Using the theory of generalized functions to model the reaction forces of the absorbers, the procedure applies in frequency and time domains. In the frequency domain, the key novelty is the derivation of the exact dynamic Green's functions under harmonic point loads. Further relevant novelties are analytical expressions for the modal response under arbitrary loads, in frequency and time domains. The procedure is general and provides elegant solutions for various engineering problems involving twisted beams, composite beams, coupled bending-torsion beams equipped with different types of absorbers.

Keywords: high-order beam, vibration absorber, generalized functions

1. Introduction

Beam structures equipped with vibration absorbers are generally modelled as one-dimensional continua coupled with point attachments. Typically, this model requires the solution of boundary value problems where the order of the differential equations depends on the beam characteristics, while generalized functions, as Dirac's deltas and/or its formal derivatives, model the reaction forces of the absorbers. In this work, we aim to propose a new and comprehensive mathematical framework providing exact solutions for high-order boundary value problems governing beams with vibration absorbers of relevant engineering interest.

First, we devise a general procedure to construct the exact dynamic Green's functions, which provide the beam frequency response to an arbitrarily placed harmonic point load. This result is based on deriving, in a concise analytical form, the solution of an arbitrary *n*-th order differential equation under Dirac's deltas and its formal derivatives. The exact dynamic Green's functions are the basis to obtain, by straightforward integration, the frequency response to arbitrarily placed harmonic distributed loads.

Next, orthogonality conditions are derived for the beam modes, which lead to analytical expressions of the modal response under arbitrary loads, in frequency and time domains. For this result, the key concepts are self-adjointness of the beam differential operators and derivation of frequency-dependent stiffness terms expressing the reaction forces of the absorbers. In this context, modes and eigenvalues are derived from a dynamic-stiffness approach, built based on the exact dynamic Green's functions.

The proposed framework encompasses a wide variety of beams, as twisted beams [1], composite beams [2] and coupled bending-torsion beams [3], the equilibrium of which is governed by high-order differential equations of various order, depending on the beam characteristics. Moreover, the framework is appropriate for different types of absorbers, as mass-spring chains or inerter-based absorbers. Viscous damping within the absorbers may be also included.

2. Results and Discussion

We consider the coupled bending-torsion beam shown in Fig. 1, equipped with a tuned mass damper (TMD) at $y=0.5L$. The proposed framework can be used to calculate, e.g., the exact dynamic Green's functions shown in Fig. 2, under a harmonic point load at $y_0=0.275L$. $H(\omega)$ and $\Psi(\omega)$ are the bending deflection and the torsional rotation. The following parameters are considered: m=7.83 kgm^{-1}, I_α=0.055 kgm, GJ=2545 Nm^2, EI=712194 Nm^2, x_a=0.051 m, L=3 m, M=20 kg, c=50 Nsm^{-1}, k=10^6 Nm^{-1}.

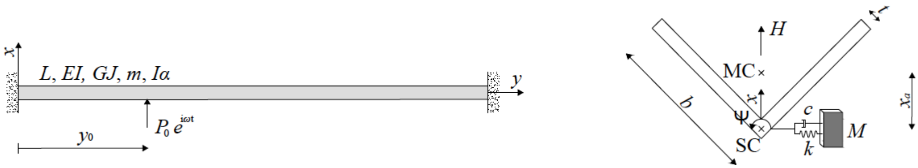

Fig. 1. Beam with V cross section equipped with a TMD and subjected to harmonic point load.

The exact dynamic Green's functions in Fig. 2 are obtained in analytical form, as solution of the sixth-order coupled bending-torsion differential equation of the beam acted upon by a Dirac's delta representing the point load and a Dirac's delta representing the reaction of the TMD.

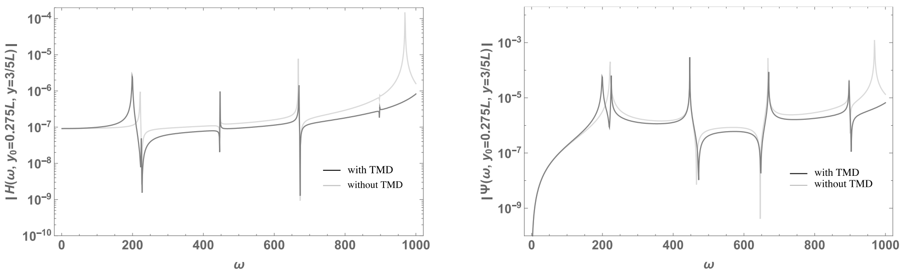

Fig. 2. Dynamic Green's functions for the beam in Fig. 1 computed with and without TMD.

3. Concluding Remarks

We propose a novel solution procedure for high-order boundary value problems of relevant interest in mechanics, involving various types of beams with different types of absorbers. The procedure applies in frequency and time domains, under arbitrary loads.

References

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