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# Influence of fractional order parameter on the dynamics of different vibrating systems

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**Abstract:** In this work, we will investigate the fractional differential equations associated to different vibration phenomena. More specifically, we will discuss Bagley-Torvik equation, composite fractional relaxation differential equation and the motion of a linear oscillator using fractional derivative operator in the sense of Atangana-Baleanu. In order to be consistent with the physical systems the value of the fractional parameter that characterizes the existence of fractional structures in the system, lies within unit interval. The solutions of the non-integer order differential equation are obtained and expressed in terms of generalized functions depending upon the fractional parameter. The classical cases could be recovered by making the limit of fractional parameter approaches to unity. Moreover, we will analyze and compare the control of the fractional order parameter on the dynamics of the models and useful conclusions are recorded.

**Keywords:** fractional derivative operator, vibrating systems, linear oscillators

## 1. Introduction

From the last few decades fractional calculus has been motivating and attracting in a great deal of consideration of researchers, physicists and mathematicians. It is seen that different interdisciplinary problems can likewise be solved with good accuracy by the aid of non integer order derivatives. In viscoelasticy, the introductory implementation of fractional calculus is seem to be done by Bagley and Torvik [1], while Makris et al. [2] approximated the applicable value of non integer order parameter that has good compliance with the experimental and material properties anticipated by non integer order derivative model. In addition, fractional order generalizations of one dimensional viscoelastic models have been found to be of great utility in displaying the response linear regime and they are in agreement with the second law of thermodynamics. So, list of the application of fractional calculus is too long to be included here.

Many physical phenomenon have inherent fractional order characterization, hence, fractional calculus is necessary to explain them. Non integer order derivative provide an excellent instrument for the explanation of memory and hereditary characteristics of various materials and processes. This is the main advantage of fractional calculus in comparison with the classical integer order models, in which such properties are in fact ignored and clearly inadequate to certify suitable correlation with experimental data.

Different investigations are made for the study of fractional oscillator equations, for example Ryabov and Puzenko [3] respectively Naber [4] investigated the fractional oscillator equation in the setting of Riemann-Liouville type and Caputo type fractional derivative operator. Stanislavsky [5] interpreted the fractional oscillator equation as the ensemble of ordinary harmonic oscillators governed by stochastic time arrow. Gaul in [6] discussed the damping description by employing fractional operators and investigated the influence of damping in waves and vibrations. It is important to note that, in the mostly published papers researchers have used Riemann-Liouville or Caputo differential operators with singular kernel. More recently, Atangana and Baleanu [7] have proposed non-singular kernel based modern definitions of fractional derivative. It possess all advantages of Caputo and Riemann-Liouville operators but have smooth kernels.

With these motivations in mind, our aim is to study the fractional differential equations associated to different vibration phenomena. More specifically, we will discuss Bagley-Torvik equation, composite fractional relaxation differential equation and the motion of a linear oscillator using fractional derivative operator in the sense of Atangana and Baleanu. A thorough investigation is made for different driving force functions and different values of fractional order parameter and useful conclusions will be recorded.

## 2. Concluding Remarks

From the solutions of the fractional Bagley-Torvik equation, it is noticed that the motion of the plate is the increasing function of the fractional parameters and influence is sensitive to the applied force to the plate. Our results could be used as the exact solutions for the comparison of the new numerical methods for the solution of the Bargley-Torvik equation. From the analysis of the solutions of fractional relaxation oscillator it is observed that for increasing values of fractional order parameter velocity increases. The velocity attains the constant value after its initiation and time to reach this constant value is smaller for large value of $\frac{\rho_p}{\rho_f}$. From the analysis of the solutions of fractional damped harmonic oscillator it is observed that the equation of oscillator in the setting of fractional derivative operators without damping term still shows damping features depending upon the fractional order parameter. Fractional oscillators with periodic forcing represents the periodic solutions and time for transients to disappear is proportional to the fractional order parameter. By quasi periodic excitations, the behaviour of the oscillator is quasi periodic but periodicity could be achieved by customizing the fractional derivative operator and its order. For the explanation of memory and hereditary characteristics of oscillator fractional derivative approach is more useful. ABC operators have non-singular kernel, so these operators are more preferable to adopt in the fractional order vibration phenomena.

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