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Transversal-transversal internal resonances in planar Timoshenko beams with an elastic support

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Abstract: The goal of the present work is to explore flexural-flexural internal resonance in an analytical model of Timoshenko beams with whatever axial boundary constraint. First, a set of partial differential equations is treated by multiple time scales method, and next selected results are compared with finite element simulations. Two way coupling between longitudinal and transverse deformations, stability analysis, detached solutions and transfer of energy are studied in depth.

Keywords: nonlinear beam, modal interactions, multiple time scales method, internal resonance

1. Introduction

We study the nonlinear dynamics of a planar, initially straight, extensible Timoshenko beam subjected to the hinged-simply supported boundary conditions and an axial elastic spring k_s at one end, see Fig. 1. Nonlinearities arise from the beam geometry as well as axial, transversal and rotary inertia of the beam, while a linear elastic behavior is assumed in the model [1]. Coupled free and forced-damped nonlinear oscillations of the primary and higher order resonances in the absence internal resonance condition have been studied by the multiple time scales method, considering quadratic and cubic nonlinearities, in [2,3]. The axial-transversal modal interaction between large amplitude flexural oscillations which in 2:1 internal resonance with the longitudinal displacement has been observed in [4].

In the present work the dynamic behavior of the beam with different modal interaction between two successive flexural modes is analyzed by using numerical and analytical approaches. The frequency of the second mode is approximately three times that of the first mode and hence a *3:1* internal resonance can be activated [4,5]. The influence of the longitudinal spring stiffness on the nonlinear system response is investigated and stability is checked by the Jacoby method.

Fig. 1. The beam-spring system

2. Results and concluding remarks

The natural frequencies of the first and second modes are independent of the axial spring stiffness k_s. Despite this fact, the fundamental resonance can be varied from softening to hardening in term of passive control. In addition, the natural frequency in the longitudinal direction trigs the secondary resonance born, which can lead to detached solution path (isolas) as well, see Fig.2. The k_s increment demonstrates secondary resonance reduction, which translates into blocking the longitudinal displacement of the beam tip (hinged-hinged beam).

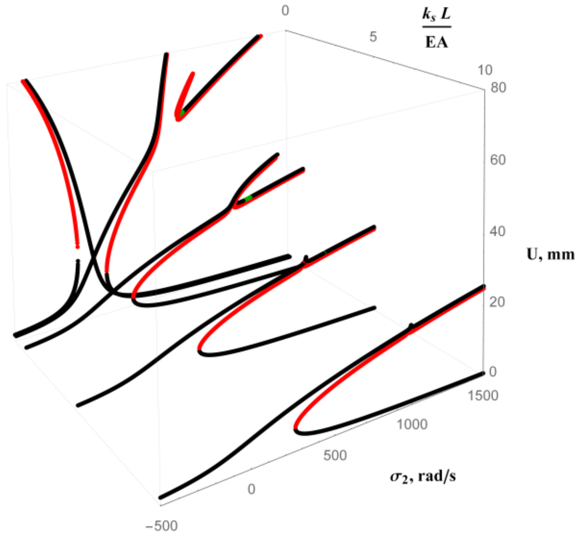

Fig. 2. Frequency response curves of the beam-spring system for *Z=1/4L* and dimensionless stiffnesses coefficients k_sL/EA equal *0, 0.1, 1, 5* and *10* (back to front). Jacoby stability analysis: stable (black), unstable saddle-type (red) and unstable source type (green) solutions.

In the future, extension of present work to experimental validation by the kinematical excitation on slip table is planned. The analytical and numerical model will be improved to consider the additional tip mass and moment of inertia at the ends.

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