Global Sliding Mode Control for a Fully-Actuated Non-Planar Hexa-Rotor Aerial Vehicle

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Abstract: This paper is concerned with the attitude and position control of a fully-actuated non-planar hexa-rotor aerial vehicle equipped with reversible fixed rotors. A six-degrees-of-freedom force-torque control law is designed using a unit-vector multi-input global sliding mode control. The method is evaluated and demonstrated in a software-in-the-loop simulator, which shows its effectiveness.
Keywords: multirotor aerial vehicle, hexa-rotor, global sliding mode control, dynamics.

1. Introduction

Applications of multirotor aerial vehicles (MAVs) for aerial manipulation, delivery and air taxi are expected in the near future. These tasks require the vehicle to safely maneuver in position independently of the attitude, while subject to unknown environmental disturbances (*e.g*, wind) [1]. Therefore, these applications are quite suitable for fully-actuated MAVs, such as the non-planar hexa-rotor aerial vehicle considered in this paper.

To allow a safe flight in the presence of bounded disturbances, we design a sliding mode controller (SMC) suitable for fully-actuated MAVs. The controller provides a six-dimensional command for the resultant force and torque, thus it can control both the vehicle's translational and rotational dynamics. The conventional SMC design consists of two phases: reaching a specified sliding manifold and sliding along this manifold [2]. However, during the reaching phase, robustness is not guaranteed. Therefore, we use the global sliding mode control (GSMC) scheme, which provides robustness from the initial condition until the desired reference [2], [3]. This abstract briefly shows the control methodology, which includes the MAV dynamic modeling, the adopted sliding surface, and the proposed GSMC law. Additionally, we present the simulation results of the proposed control compared to a proportional-derivative (PD) feedback linearization control law, considering a non-planar hexa-rotor aerial vehicle.

2. Controller Design and Results

Assume that $\mathbf{x} \triangleq (\mathbf{x}_1, \mathbf{x}_2) \in \mathbb{R}^{12}$ is the vector of state errors, where $\mathbf{x}_1 \in \mathbb{R}^6$ denotes the errors of position and attitude, which is expressed as a Gibbs vector, and $\mathbf{x}_2 \in \mathbb{R}^6$ represents the errors of linear and angular velocities. Moreover, consider the control vector $\mathbf{u} \in \mathbb{R}^6$ as a command to the force and torque produced by the MAV, while $\mathbf{d} \in \mathbb{R}^6$ is an unknown force-torque disturbance such that $\|\mathbf{d}\| \leq \rho$, with known $\rho \in \mathbb{R}_+$. Therefore, the complete nonlinear dynamics of the MAV can be modelled as follows

$$\dot{\mathbf{x}}_1 = \mathbf{f}_1(\mathbf{x}), \tag{1}$$

$$\dot{\mathbf{x}}_2 = \mathbf{f}_2(\mathbf{x}) + \mathbf{B}(\mathbf{x})\,(\mathbf{u} + \mathbf{d}), \tag{2}$$

where $\mathbf{f}_1 : \mathbb{R}^{12} \to \mathbb{R}^6$, $\mathbf{f}_2 : \mathbb{R}^{12} \to \mathbb{R}^6$, and $\mathbf{B} : \mathbb{R}^{12} \to \mathbb{R}^{6\times 6}$ are given functions. Furthermore, $\mathbf{f}_1$ and $\mathbf{B}$ are such that $\|(\partial \mathbf{f}_1/\partial \mathbf{x}_2)\mathbf{B}\| \neq 0, \forall \mathbf{x} \in \mathbb{R}^{12}$. Now, we can define the following time-varying sliding function

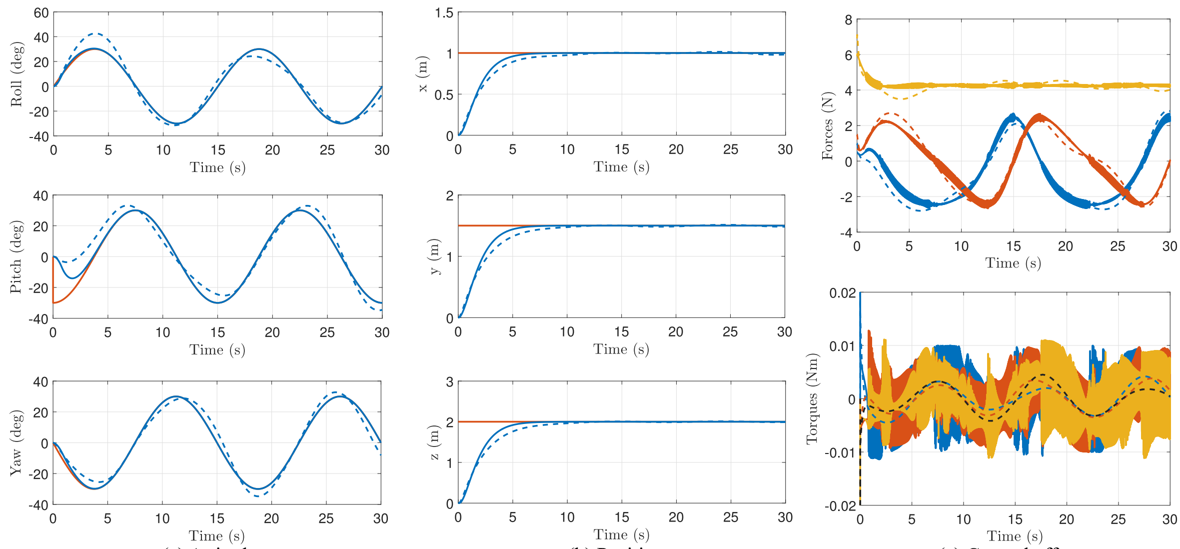
$$\mathbf{s}(t,\mathbf{x}) \triangleq \boldsymbol{\sigma}(t) - \mathbf{P}(t)\boldsymbol{\sigma}(0), \tag{3}$$

where $\boldsymbol{\sigma}(t) \triangleq \mathbf{C}\mathbf{x}_1 + \mathbf{f}_1(\mathbf{x}) \in \mathbb{R}^6$, with $\mathbf{C} \in \mathbb{R}^{6\times 6}$ being a given diagonal matrix, and $\mathbf{P} : \mathbb{R}_+ \to \mathbb{R}^{6\times 6}$ is a function which satisfies, among other design conditions, $\mathbf{P}(0) = \mathbf{I}_6$ [3]. Based on (3), consider the set $\mathscr{S} \triangleq \{\mathbf{x} \in \mathbb{R}^{12} : \mathbf{s}(t,\mathbf{x}) = 0, \forall t \geq 0\}$ as the eventual sliding set. Therefore, to ensure a global sliding mode of system (1)–(2) in $\mathscr{S}$, we can design the control law [3]

$$\mathbf{u} = -\left(\frac{\partial \mathbf{f}_1}{\partial \mathbf{x}_2}\mathbf{B}\right)^{-1}\left(\left(\mathbf{C} + \frac{\partial \mathbf{f}_1}{\partial \mathbf{x}_1}\right)\mathbf{f}_1 + \frac{\partial \mathbf{f}_1}{\partial \mathbf{x}_2}\mathbf{f}_2 + \dot{\mathbf{P}}(t)\boldsymbol{\sigma}(0) + \kappa\frac{\mathbf{s}}{\|\mathbf{s}\|}\right), \tag{4}$$

where $\kappa > \|(\partial \mathbf{f}_1/\partial \mathbf{x}_2)\mathbf{B}\|\rho$ is a design parameter.

Figure 1 shows the controlled position and attitude, as well as the respective commands. The reference trajectory is a step of $(1, 1.5, 2)^{\mathrm{T}}$ for position, combined with a conic motion with frequency of $1/15$ Hz and amplitude of 30° for attitude. The disturbances are considered as sinusoidal signals with frequency of 0.1 Hz and amplitudes of 0.02 N and 0.003 Nm for force and torque, respectively. The parameters used for the GSMC are $\kappa = 0.5$, $\mathbf{C} = \mathbf{I}_6$, and $\mathbf{P}(t) = \exp(-t)\mathbf{I}_6$. For the PD control, $\mathbf{K}_1 = \mathrm{diag}(2,\ 2,\ 2,\ 3,\ 3,\ 3)$ is the proportional gain and $\mathbf{K}_2 = 5\mathbf{I}_6$ is the derivative gain. For the position states, we can note that the GSMC does not present overshoot neither oscillations around the commanded value. On the other hand, for the attitude states, the GSMC reaches the reference faster and follows it precisely.



(a) Attitude (b) Position (c) Control efforts

Figure 1: Numerical results for attitude and position control of a fully-actuated non-planar hexa-rotor aerial vehicle, where for attitude and position - - - PD control, — GSMC, and — the command trajectory. For the GSMC — T_1, — T_2, — T_3 the control torques and — F_1, — F_2, — F_3 the control forces. For the PD control, - - - T_1, - - - T_2, - - - T_3 the control torques, and - - - F_1, - - - F_2, - - - F_3 the control forces.

3. Concluding Remarks

This paper has presented the design of a global sliding mode control law for a fully-actuated non-planar hexa-rotor and has compared the results with a proportional-derivative feedback linearization control law. We can observe that the GSMC outperforms the PD control, while ensuring robustness against bounded disturbances during all the flight.

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