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Control of micorgrid synchronization based on feedback control and optimzation techniques

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Abstract: In this paper, we propose a secondary feedback control of a droop-controlled smart grid, modelled via the first order Kuramoto model. We show that we can improve the transient evolution of the system by applying a harmonic secondary control on each oscillator which only makes use of the error between the node's state and the desired equilibrium state. We show in a 2 dimensional coupled oscillator model that, while the effect of this control action is to move the spectrum of the eigenvalues, the topology of the system imposes restrictions over the admissible eigenvalues that need to be preserved in the controlled system. To overcome this, we find the set of gains that fulfills the restriction over one eigenvalue while allowing to move the second one to a desired value, achieving improved transient dynamics in terms of settling times.

Keywords: Synchronization, Kuramoto oscillators, micrograms, feedback control

1. Introduction

A droop-controlled smart grid can be represented as a First Order Kuramoto model [1,2]

$$\dot{\theta}_i(t) = \omega_i + K\sum_{j}^{N} A_{ij}\sin\left(\theta_j(t) - \theta_i(t)\right) \tag{1}$$

Power sharing conditions require $\sum_i \omega_i = 0$. This guarantees that synchronization is reduced to the study of stability of the equilibrium $\boldsymbol{\theta}^*$. An approximation of the equilibrium point as: $\boldsymbol{\theta}^* = \frac{L^\dagger \boldsymbol{\omega}}{K}$. In this equation, L is the Laplacian matrix and $L^\dagger$ denotes the Moore-Penrose pseudoinverse. Linearizing Eq. (1) around the equilibrium leads to the Jacobian matrix:

$$J_{ij} = \begin{cases} -K\sum_{j\neq i} A_{ij}\cos(\theta_j^* - \theta_i^*) & \text{if } i = j \\ KA_{ij}\cos(\theta_j^* - \theta_i^*) & \text{otherwise} \end{cases} \tag{2}$$

Following [3], a secondary feedback control can be applied in the following form:

$$\dot{\theta}_i(t) = \omega_i + K\sum_{j}^{N} A_{ij}\sin(\theta_j(t) - \theta_i(t)) + F_i\sin(\theta_i^* - \theta_i).$$

Here, F_i acts as the gain of the controller for the i-th oscillator and modifies the diagonal entries of the Jacobian matrix, as $J_{ii} = -K\sum_{j\neq i} A_{ij}\cos(\theta_j^* - \theta_i^*) - F_i$. We propose the application of this type of feedback control, to modify the dynamics of the phase oscillator model , by placing the eigenvalues of

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the system according to a desired design criteria, taking into account that one eigenvalue must remain on the origin, in order to preserve the rotational degeneracy of phase oscillator. With these ideas in mind, the solutions of the gains in terms of the desired controlled eigenvalue $\lambda_2^{(c)}$ are:

$$F_{1,2} = -\frac{1}{2}\left(\lambda_2^{(c)} + \lambda_2^{(u)} \pm \sqrt{-\lambda_2^{(c)} - \lambda_2^{(u)}}\right) \tag{3}$$

In the above equation the superscripts superscripts *(c)* and *(u)* stand for controlled or uncontrolled eigenvalues. Recall that, due to the restrictions $\lambda_1^{(c)}$ is strictly equal to zero.

2. Results and discussion

Figure 1(a) shows the solutions of F_i as a function of the desired eigenvalue $\lambda_2^{(c)}$. It is important to notice that $F_1 = F_2 = 0$ corresponds to the uncontrolled system, therefore the controlled eigenvalue is always smaller than the uncontrolled one. Fig 1(b) shows the improved dynamics of the controlled system (red) compared with the uncontrolled one (black). This can be seen with the smaller settling time of the order parameter, quantifying the degree of synchronization between the oscillators (top panel). It is also possible to observe that the system reaches the same steady value as the uncontrolled case as expected, since the control does not vary $\boldsymbol{\theta}^*$.

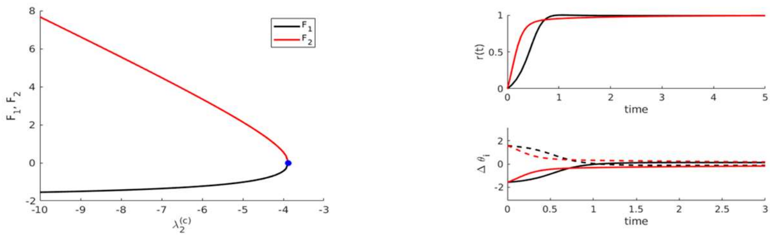

Fig. 1. Left: Gains as a function of the desired eigenvalue given by Eq. (3) for the two oscillator system. For this graphic, $\omega_1 = ½$, ω_2= -½, and $K = 2$. **Right**: Dynamics of the controlled system (red) compared with the uncontrolled system (black). **Top**: the order parameters is depicted. **Bottom**: The error between the values of the phases and its steady value**.**

3. Concluding Remarks

The secondary control loop was successful to improve the performance of the controlled system. The transient response showed a faster dynamics than the uncontrolled one and we obtained an analytical expression to calculate the feedback gains. We propose to extend the algorithm to higher order systems which requires the implementation of optimization routines subject to restrictions. This will be the topic of the extended version of this manuscript.

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