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On Qualitative Analysis of the Model of Two-Link Manipulator with Time Delays: Stability, Bifurcation and Transition to Chaos

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Abstract: The paper considers the model of a two-link manipulator with constant delays. For the given reference trajectory the model is reduced to the four-order system of ordinary differential equations with two delays, which allows us to apply the methods of qualitative analysis of dynamics' systems. Stability research focuses on local asymptotic stability with the help of analyzing quasipolynomial near equilibrium state. In one special unstable case, the eigenvalues in a clear form are gotten. Nonlinear analysis is based on phase plots and computing some numerical characteristics. Namely, the bifurcation plot which was constructed on the base of the Poincare section has shown the appearance of chaotic behavior as a result of increasing time delays. The analytical results of the work include presenting the model in the form of ordinary differential equations with delays, finding eigenvalues of the characteristic polynomial in some special case. They have been obtained with the help of symbolic computation in the Yacas computer algebra system. Numerical characteristics with the facilities of visualization have been calculated in nonlinearTseries package in R.

Keywords: two-link manipulator, local asymptotic stability, delay, nonlinear analysis, Poincare section, bifurcation plot.

1. Introduction

Recently the model of a two-link manipulator (Fig. 1) attracts more attention from viewpoint of computing facilities of software to get its trajectories [1-3]. On the other hand, its qualitative analysis is of importance. In turn, cumbersome nonlinearities in the right-hand side do not allow us to get clear analytical results, e.g., for stability research. Here we study the delayed model on the basis of the system of nonlinear differential equations with angles θ_1, θ_2 and velocities

$$z_1 = \frac{\mathrm{d}\theta_1}{\mathrm{d}t},\ z_2 = \frac{\mathrm{d}\theta_2}{\mathrm{d}t}. \qquad (1)$$

Thus, given reference trajectory $\left(\theta_1^r(t), \theta_2^r(t), z_1^r(t), z_2^r(t)\right)$ we consider the system

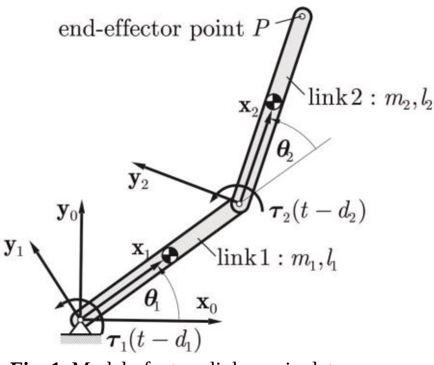

Fig. 1. Model of a two-link manipulator

$$\tau_1(t-d_1) = \left(\frac{m_1 l_1^2}{3} + m_2 l_1^2 + \frac{m_2 l_2^2}{3} + m_2 l_1 l_2 \cos\theta_2\right)\frac{\mathrm{d}z_1}{\mathrm{d}t} + \left(\frac{m_2 l_2^2}{3} + \frac{m_2 l_1 l_2}{2}\cos\theta_2\right)\frac{\mathrm{d}z_2}{\mathrm{d}t} - \frac{m_2 l_1 l_2}{2}\sin\theta_2\, z_2^2 - m_2 l_1 l_2 \sin\theta_2\, z_1 z_2 + \frac{m_1 g l_1}{2}\cos\theta_1 + \frac{m_2 g l_2}{2}\cos(\theta_1+\theta_2) + m_2 g l_1 \cos\theta_1,$$

$$\tau_2(\mathrm{t}-\mathrm{d}_2) = \left(\frac{m_2 l_2^2}{3} + \frac{m_2 l_1 l_2}{2}\cos\theta_2\right)\frac{\mathrm{d}z_1}{\mathrm{d}t} + \frac{m_2 l_2^2}{3}\frac{\mathrm{d}z_2}{\mathrm{d}t} + \frac{m_2 l_1 l_2}{2}\sin\theta_2\, z_1^2 + \frac{m_2 g l_2}{2}\cos(\theta_1+\theta_2), \qquad (2)$$

where

$$\tau_1(t-d_1) = k_{1,1}^1\big(\theta_1(t-d_1) - \theta_1^r(t-d_1)\big) + k_{1,2}^1\big(z_1(t-d_1) - z_1^r(t-d_1)\big) + k_{1,1}^2\big(\theta_2(t-d_2) - \theta_2^r(t-d_2)\big) + k_{1,2}^2\big(z_2(t-d_2) - z_2^r(t-d_2)\big),$$

$$\tau_2(t-d_2) = k_{2,1}^1\big(\theta_1(t-d_1) - \theta_1^r(t-d_1)\big) + k_{2,2}^1\big(z_1(t-d_1) - z_1^r(t-d_1)\big) + k_{2,1}^2\big(\theta_2(t-d_2) - \theta_2^r(t-d_2)\big) + k_{2,2}^2\big(z_2(t-d_2) - z_2^r(t-d_2)\big)$$

m_1, m_2, l_1, l_2 are masses and lengths of links, g is acceleration of gravity, $k_{i,j}^n, i, j, n \in \{1,2\}$ are gains, d_1, d_2 are constant time delays with the respect to the first and second joints respectively. Equations (2) can be presented as nonstationary ordinary differential equations with the delays

$$\frac{\mathrm{d}z_1}{\mathrm{d}t} = f_1\big(t, \theta_1(t), \theta_2(t), z_1(t), z_2(t), \theta_1(t-d_1), \theta_2(t-d_2), z_1(t-d_1), z_2(t-d_2)\big),$$
$$\frac{\mathrm{d}z_2}{\mathrm{d}t} = f_2\big(t, \theta_1(t), \theta_2(t), z_1(t), z_2(t), \theta_1(t-d_1), \theta_2(t-d_2), z_1(t-d_1), z_2(t-d_2)\big). \qquad (3)$$

That is (1), (3) combined with the corresponding initial conditions present model to be investigated.

2. Stability Research

In the non-delayed case when investigating local asymptotic stability, we have calculated Jacobian for the system (1), (3) for the neighbourhood of equilibrium state $\mathcal{E}_1^0 = (0,0,\theta_1^0,\theta_2^0)^\top$ having the form

$$J = \begin{bmatrix} 0 & 0 & J_{13} & J_{14} \\ 0 & 0 & J_{23} & J_{24} \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \qquad (5)$$

and corresponding eigenvalues being

$$\lambda_{1,2,3,4} = \pm\sqrt{\frac{1}{2}\left(J_{13} + J_{24} \pm \sqrt{(J_{13}+J_{24})^2 - 4(J_{14}J_{23} - J_{13}J_{24})}\right)},$$

References

[1] AHMED, S., WANG, H., ASLAM, M. S., GHOUS, I., & QAISAR, I. (2019). Robust Adaptive Control of Robotic Manipulator with Input Time-varying Delay. *International Journal of Control, Automation and Systems.* DOI:10.1007/S12555-018-0767-5

[2] PUSHKAL B., JOSY G.: Two Link Planar Robot Manipulator Mechanism Analysis with MATLAB. *International Journal for Research in Applied Science & Engineering Technology* 2018, **6**(VII):778-788.

[3] OMID MEHRJOOEE, SIAVASH FATHOLLAHI DEHKORDI & MOHARAM HABIBNEJAD KORAYEM (2019): Dynamic modeling and extended bifurcation analysis of flexible-link manipulator, *Mechanics Based Design of Structures and Machines*, DOI: 10.1080/15397734.2019.1665542