16th INTERNATIONAL CONFERENCE
Dynamical Systems – Theory and Applications
December 6-9, 2021, On-line

On the dynamics of a 2-DOF nonlinear vibratory system with bistable characteristic and circulatory forces

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Abstract: We consider an autonomous damped 2-DOF mechanical system in which the two DOFs are coupled by a linear spring. A circulatory force is introduced into the system that may result in self-excited vibrations. A nonlinearity is added in the form of a cubic spring so that there are three fixed points, two of them are stable without the circulatory force, i.e., a bistable behaviour. The basins of attraction for different values of the circulatory force are numerically studied to see how the patterns of them change. Using a Poincaré map, the basins of attraction have three dimensions and they are cut with further cross-sections for the ease of visualization. The results are usually complicated maps with non-smooth boundaries, except when the circulatory force uncouples one DOF from the other. Special patterns of the maps are seen when in the nonlinear case a stable limit cycle is about to occur. Even in the case without stable limit cycle, initial conditions within the special range may lead to hundreds of cycles of periodic-like transient motion before asymptotically converge to a stable fixed point. Phase planes and bifurcation diagrams are also used, and multiple coexisting periodic solutions are found. Strong symmetric and asymmetric autonomous bursts are found.

Keywords: basins of attraction, circulatory force, self-excited vibration, transient motion, bursting

1. Introduction

Mechanical systems with circulatory terms show a number of interesting phenomena, e.g., transient growth [1]. In an earlier paper [2], the authors investigated differences in the behaviours of linear 2-DOF systems with and without circulatory matrices having same maximum real parts of eigenvalues. The current presentation is adding the influence of nonlinearities and focusing on the circulatory case. The added nonlinearity is in the form of a cubic spring with a negative linear term which is adjusted according to the circulatory term so that there are two fixed points $(x, z) = (1,0.1)$ and $(-1,-0.1)$ aside from the unstable trivial solution. Phase planes, basins of attraction and bifurcation diagrams are used to study the global dynamics of the system. Multiple numerical methods, including cell-mapping method, are employed.

2. Results and Discussion

When the circulatory term (characterized by parameter χ) is small enough, solutions are converging to one of the two stable fixed points. The basins of attraction for different values of χ are numerically studied to see how the patterns of them change. The results are maps with non-smooth boundaries, except when the circulatory force uncouples one DOF from the other (Fig. 1a). The cross-sections of the basins of attraction show partially structured as well as fractal pattern (Fig. 1b) when χ increases to a bifurcation value where the first inter-well limit cycle appears. The basin of attraction of the limit cycle appears and grows within the region of that special pattern (Fig. 1c). Closely before the bifurcation value, the limit cycle does not exist yet, but a periodic-like transient motion lasting for hundreds of cycles is observed.

When χ is high enough, all the fixed points become unstable, and many more periodic solutions appear, both symmetric and asymmetric. The periodic solutions show stronger bursting characteristic when χ is increased. Coexisting stable periodic solutions and period-doubling bifurcations are seen on the bifurcation diagram.

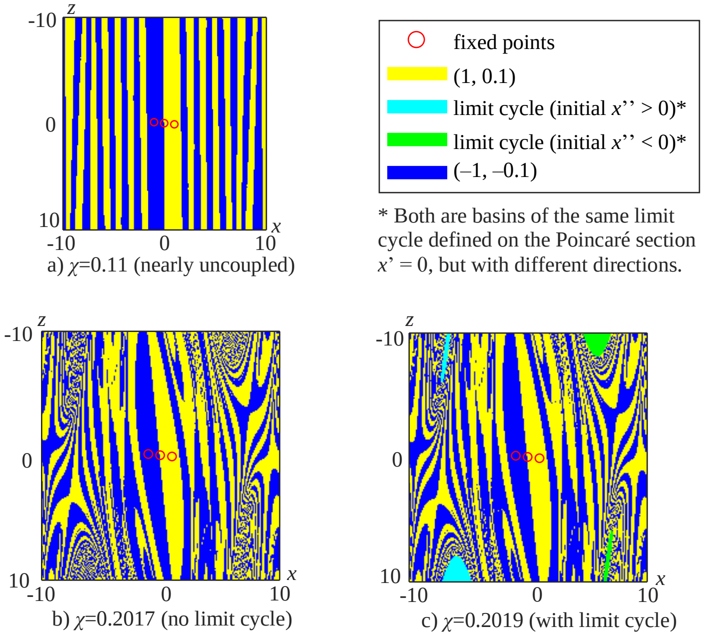

Fig. 1. Basins of attraction

3. Concluding Remarks

The circulatory force strongly affects the system's behaviour both in the linear case [2] and in the nonlinear case. Special patterns of basins of attraction are seen when in the nonlinear case the first inter-well limit cycle nearly occurs. The considered system also witnesses a long-lasting transient motion, which might not be negligible in practical applications. Many more coexisting stable periodic solutions are found. The solutions are strongly bursting with appropriate parameters.

References

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