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Chaos and Bifurcations in a Nonlinear Dynamics of Chain of the Backward-Wave Tubes: Numerical Analysis

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Abstract: The paper is devoted to studying dynamical characteristics of non-linear processes in the chain of non-relativistic and relativistic bachward-wave tubes (BWT) and modeling parameters for the corresponding chaotic time series, which are the solutions of the BWT integral-differential dynamical equations. The computational chaos-geometric approach includes a combined set of non-linear analysis and chaos theory methods such as an autocorrelation function method, correlation integral approach, average mutual information, surrogate data, false nearest neighbours algorithms, the Lyapunov's exponents and Kolmogorov entropy analysis, spectral methods and nonlinear prediction (predicted trajectories, neural network etc) algorithms. There are computed the dynamic and topological invariants in auto-modulation/chaotic regimes. The bifurcation diagrams in the plaines of different governing parameters are constructed.

Keywords: chain of backward-wave tubes, chaos, bifurcations, attractors

1. Introduction.

It is well known that the back-ward tube (BWT) is an electronic device for generating electro-magnetic vibrations in the super high frequencies range. The intensive theoretical studies (e.g. [1-3]) have been performed for were of dynamics of a non-relativistic BWT, in particular, phase portraits, statistical quantifiers for a weak chaos arising via period-doubling cascade of self-modulation and the same characteristics of two non-relativistic backward-wave tubes. The differential equations of non-stationary nonlinear theory for the O-type BWT without account of the spatial charge, relativistic effects, energy losses etc have been solved. It has been shown that the finite-dimension strange at-tractor is responsible for chaotic regimes in the BWT.

In our paper the advanced results of studying dynamical characteristics of non-linear processes in the chain of non-relativistic and relativistic BWTs are presented. It has been perfomed a detailed analysis and modeling the parameters of the corresponding chaotic time series for the charateristic amplitudes, which are the solutions of the BWT integral-differential dynamical equations. The computational chaos-geometric approach includes a combined set of non-linear analysis and chaos theory methods such as an autocorrelation function method, correlation integral approach, average mutual information, surrogate data, false nearest neighbours algorithms, the Lyapunov's exponents and Kolmogorov entropy analysis, spectral methods and nonlinear prediction (predicted trajectories, neural network etc) algorithms (in versions [3-5]). A dynamics of the chain of two backward-wave tubes is in deatils considered. There are computed the dynamic and topological invariants in auto-modulation/chaotic regimes.

2. Dynamics of backward-wave tubes chain: Results and Discussion

A chain of two relativistic BWTs can be described by the standard master system of evolutionary differential equation as follows:

$$\partial^2\theta_{1,2}/\partial\xi^2 = -(1+\nu\,\partial\theta/\partial\xi)^{3/2}\mathrm{Re}\{{}^1/_2\,L_{1,2}[\delta(\xi)+\delta(\xi-L)F_{1,2}e^{i\theta_{1,2}}]\},$$
$$\partial F_{1,2}/\partial\tau - \partial F_{1,2}/\partial\xi = -\left[1/\pi\int_0^{2\pi}e^{-i\theta_{1,2}}d\theta_0\right]{}^1/_2\,L_{1,2}[\delta(\xi)+\delta(\xi-L_{1,2})]. \quad (1)$$

where $\theta_{1,2}$ are the phases of electron relative to the wave, θ_0 - the initial phase, $F_{1,2}$ are the dimensionless slowly varying amplitudes of fields { $E(x,t)=\mathrm{Re}[E(x,t)\exp(i\omega_0 t-i\beta_0 x)]$ }, $\xi==\beta_0 Cx$ and τ are the dimensionless coordinate and time, respectively; parameter $L=\beta_0 lC=2\pi CN$ is the dimensionless length of the interaction space, l is a length of the system, N is a number of slow waves, covering over the length of system, $C=\sqrt[3]{I_0K_0/(4U)}$ is the known Pierce parameter, I_0 is a current of beam, U is an accelerated voltage, K_0 is a resistance of link of the slowing system, ν is the known relativistic parameter, C- modified gain parameter. The first equation in the system (1) represent equation of motion of electrons in the field of the electromagnetic wave and the second equation is the non-stationary equations of excitation of a decelerating structure by a current of the slowly varying amplitude. The subscripts indicate the item number of the chain. The dynamics of the partial generator depends on a single bifurcation parameter $L=2\pi CN$. Two different modes of operation are considered. In the first case it is supposed that the BWTs are operating in regime of the periodical automodulation. The values of the L parameter are: L_1=4.05, L_2=4.55. In the first case one deals with the Feigenbaum type scenario. In the second case there is the transition “chaos-order” through intermittency that is confirmed by the corresponding chaos-geometric numerical analysis data.

3. Concluding Remarks

Study of the dynamic characteristics of non-linear processes in the chain of non-relativistic and relativistic BWTs is perfoemed and modeling the dynamic characteristics for the corresponding chaotic time series is carried out. The new data on the dynamic and topological invariants of the BWT chain dynamics in auto-modulation/chaotic regimes are obtained. The bifurcation diagrams in the plaines of different governing parameters are constructed.

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