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Nonlinear Dynamics of Relativistic Backward-Wave Tube: Chaos, Bifurcations and Strange Attractors

ALEXANDER V GLUSHKOV[1*], ANDREY V TSUDIK[1], OLEG V DUBROVSKY[1] AND OLEKSII L MYKHAILOV[1]

1. Odessa State Environmental University, Mathematics Depr., L'vovskaya str. 15, 65009, Odessa
* Presenting Author

Abstract: The paper is devoted to modelling, analysis, forecasting the dynamics of relativistic backward-wave tube (RBWT) with accounting for relativistic effects (factor γ_0), dissipation factor (factor D) and an effect of presence of the space charge. The temporal dependences of the normalized field amplitudes (power) in a wide range of variation of the governing parameters (electric length of an interaction space, bifurcation parameter, the Pirse parameter and γ_0) are computed and analyzed from the viewpoint of distributed relativistic electron-waved self-vibrational systems. A nonlinear analysis technique (including a multi-fractal approach, the methods of correlation integral, false nearest neighbours, surrogate data, the Lyapunov's exponent's and Kolmogorov entropy algorithms etc) is applied to numerical studying the RBWT chaotic dynamics. There are computed the dynamic and topological invariants in auto-modulation/chaotic regimes. The bifurcation diagrams in the plaines of different governing parameters are constructed.

Keywords: relativistic backward-wave tube, chaos, attractors

1. Non-stationary dynamics of relativistic backward-wave tube: Master system of evolutionary equations and nonlinear analysis

At present, a study of regular and chaotic dynamics of nonlinear processes in different classes of devices of so-called relativistic high-frequency or even ultrahigh-frequency microwave electronics is of a great importance (e.g.[1-4]). Despite intensive study of the phenomenon of chaos in the BWT dynamics, now it has been recognized that many features of the self-modulation regimes, primarily chaotic ones, remain unexplored. Moreover, there are no definite answers to the question on the mechanisms of chaos generation, chaotic self-modulation onset. There are absent the quantitative data on the the dynamic and topological invariants of the RBWT dynamics. In this work the results of modelling, analysis, forecasting the RBWT dynamics with accounting for relativistic effects, dissipation factor and an effect of presence of the space charge are presented. The time dependences of the normalized field amplitudes (power) are computed in a wide range of variation of the governing parameters: relativistic factor $\gamma_0 = (1-\beta_0^2)^{-1.2}$ (where $\beta_0 = v_0/c$, v_0 is the initial velocity of the electrons), electric length of an interaction space N) and a bifurcation parameter: $L = 2\pi CN/\gamma_0$. Here the Piers parameter C is as follows: $C = \sqrt[3]{I_0 K_0/(4U)}$, where I_0 is a constant component of the beam current, U is an accelerating voltage, and K_0 is a communication resistance of the deceleration system. The equation in the usual dimensionless form for a phase $\theta(\zeta,\tau,\theta_0)$ of relativistic electron (that flew into the space of interaction with the phase θ_0 and has a coordinate ζ at time τ) and a

complex amplitude $F(\zeta,\tau)=\tilde{E}/\left(2\beta_0 UC^2\right)$ ($E(x,t)=\mathrm{Re}[\varepsilon(x,t)\exp[i\omega_0 t-i\beta_0 x)]$) are as follows:

$$\partial^2\theta/\partial\zeta^2=-L^2\gamma_0^3[(1+\frac{1}{2\pi N}\partial\theta/\partial\zeta)^2-\beta_0^2]^{3/2}Re[F\,exp(i\theta)]$$
$$\partial F/\partial\tau-\partial F/\partial\zeta=-L\tilde{I}\ ,\ \tilde{I}=-\tfrac{1}{\pi}\textstyle\int_0^{2\pi}e^{-i\theta}\,d\theta_0 \qquad (1)$$

with corresponding boundary & initial conditions. Further the methods of a chaos theory and a non-linear analysis technique (such as a multi-fractal approach, methods of correlation integral, false nearest neighbours, surrogate data, the Lyapunov's exponent's and Kolmogorov entropy algorithms etc; eg [5]) are applied to numerical analysis of the corresponding time series of the RBWT dynamics.

2. Results and Discussion

As input data, there are used the parameters: energy of electrons - 150keV, starting current of 7A composed impedance connection 0,5Ω , length of interaction space - 0,623m; other parameters are described in [1,4]. The computed temporal dependence of the RMBT power are received for the diferent injection currents. At current 7A it is set stationary mode that with increasing value of current strength transited to the periodic automodulation (I = 30A, on our data, the period of T_a = 7.3ns; experimental value [1]: 8ns), and then when I = 55A it is realized the chaotic auto-modulation mode (Fig 1a). By increasing an current to 75A there is the quasi-periodical auto-modulation (period 13.8 ns) and, finally, when the current value is more than 100A it is realized essentially chaotic regime. Note that reset of the quasi-periodic auto-modulation mode can be explained by an effect of space charge. The obtained results are compared wth the similar theoretical estimates (however without the dissipation effect) and experiment data by Ginsburg et al [1]. Further at first the results of computing a set of the dynamical and topological invariants (correlation and embedding dimensions, Lyapunov's exponents, Kaplan-Yorke dimension (d_L), and the Kolmogorov entropy, etc) are listed.

3. Concluding Remarks

The nonlinear analysis technique (including a multi-fractal approach, the methods of correlation integral, false nearest neighbours, surrogate data, the Lyapunov's exponent's algorithm and others) is applied to analysis of numerical parameters of the RBWT chaotic dynamics. The new data on the dynamic and topological invariants of the RBWT dynamics in auto-modulation/chaotic regimes are listed for the first time. The bifurcation diagrams in the plaines of different governing parameters are constructed.

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