2021 DSTA dys-ta.com
16th INTERNATIONAL CONFERENCE
Dynamical Systems – Theory and Applications
December 6-9, 2021, On-line

Zip Bifurcation in PWSC Systems

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Abstract: This communication extends the concept of zip bifurcation introduced by Miklos Farkas in 1984 in a smooth dynamical system, describing the competition de two predator species for a single regenerating prey species, to a set of PieceWise-Smooth Continuous (PWSC) dynamical systems, that have a continuous set of equilibria, with a discontinuous Jacobian. It also shows a strategy for dynamics classification based on the eigenvalues, for the case which the local two-dimensional invariant manifolds of the system exist. Analysis on the dynamic and asymptotic behavior is obtained through the real and imaginary components of the eigenvalues, associated to the system linearity along its equilibria set. We choose a criterion of geometric classification of bifurcation that preserves information about the stability, topology of the invariant set and the geometry of node and focus on a neighborhood of the isolated hyperbolic equilibrium points. Based on the results obtained in the analysis we show that the zip bifurcation discovered by Farkas is part of a more complex phenomenon. It includes the combination of two geometrical bifurcations caused by simultaneous action of the real and imaginary components of the eigenvalues associated to the system linearity along its equilibria set. The complete bifurcation scenario includes 142 geometric zip bifurcations for the class of PWSC systems studied.

Keywords: Hopf Bifurcation, Zip Bifurcation, k- and r-strategists, PWSC systems.

1. Introduction

In ecology, in studying population dynamics, it is important to know under which ecosystem conditions the coexistence of closely related species is possible or under what circumstances the principle of competitive exclusion acts (as Harden called it in 1940). The previous principle is also known as Gause's principle, in honor of the Russian biologist who observed it in 1932 in the separation of species in experimental crops. Hutchinson and Deevey state that the Gause hypothesis or principle of competitive exclusion is one of the most significant advances in theoretical ecology and one of the foundations of modern ecology. The principle of competitive exclusion is considered as one of the primary mechanisms for the process of natural selection and, therefore, the origin and evolution of species through both intraspecific and interspecific competition.

Specifically, in this work, it is shown that the occurrence of Hopf and zip bifurcations can be extended to a class of Continuous PWS nonlinear systems, which have a continuous set of equilibria. Along the length of the continuous set, the Jacobian matrix of the system is discontinuous and satisfies the Butler-Farkas conditions [1]. We present a strategy for the demonstration of the existence and classification of the so-called PWSC-Zip bifurcation. We base our approach on studying the dynamics of the eigenvalues of the linearization of both vector fields along the set of equilibria. The previous procedure is done for the case in which the PWSC system preserves each subsystem's local two-dimensional invariant manifolds that cross-intersect the equilibrium segment. When the commutations destroy the local two-dimensional invariant manifolds, it is shown that even the phenomenon of loss of attractiveness is preserved for the equilibrium segment. In this case, the stability of the interior points of the equilibrium segment cannot be determined by linearization.

2. Results and Discussion

Four numerical examples of PWSC models are built up using Mathematica software. The models preserve the two-dimensional local invariant manifolds. Four more examples of PWSC models that maintain only the attractiveness of the equilibrium segment are also created. They represent natural and artificial models of algebraic exponential type and generalize the Gilpin logistic growth model for the prey reproduction rate and the Holling III and Rosenzweig type models for the functional response of the predator. The models were found to satisfy the necessary conditions proposed by Butler and Farkas in each of the subsystems of the PWSC system and the compatibility conditions. As a consequence of the above, the Hsu et al. model is generalized to PWSC systems in the case studied by Wilken [2] and Farkas [3], that is, for the three-dimensional case.

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